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A Tale of Two Vectors

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ABSTRACT

Why (according to classical physics) do forces compose according to the parallelogram of forces? This question has been controversial; it is one episode in a longstanding, fundamental dispute regarding which facts are not to be explained dynamically. If the parallelogram law is explained statically, then the laws of statics are separate from and (in an important sense) "transcend" the laws of dynamics. Alternatively, if the parallelogram law is explained dynamically, then statical laws become mere corollaries to the dynamical laws. I shall attempt to trace the history of this controversy in order to identify *what it would be* for one or the other of these rival views to be correct. I shall argue that various familiar accounts of natural law (Lewis's Best System Account, laws as contingent relations among universals, and scientific essentialism) not only make it difficult to see what the point of this dispute could have been, but also improperly foreclose some serious scientific options. I will sketch an alternative account of laws (including what their necessity amounts to and what it would be for certain laws to "transcend" others) that helps us to understand what this dispute was all about.

1. A forgotten controversy in the foundations of classical physics

Today's classical physics textbooks tell us without ceremony that forces, as vectors (i.e. directed quantities), combine by "vector addition". A force applied at a point can be represented by an arrow from that point in the force's direction with a length proportional to the force's magnitude (see figure 1a). The resultant of forces *F* and *G* acting together at a point is a force represented by the arrow from that point forming the diagonal of the parallelogram whose adjacent sides represent *F* and *G* (figure 1b). Accordingly, this principle is frequently called "the parallelogram of forces".

Despite the routine treatment it receives today, the parallelogram of forces was the subject of considerable controversy throughout the nineteenth century. Its truth was unquestioned. The controversy concerned its explanation (that is to say, *why* it is true) and also, I shall argue, its metaphysical status. An adequate metaphysics of natural law should help us to understand what this dispute was about. Presumably, it should also leave room for any side of this scientific dispute to have possibly been correct. That is, philosophy alone should not suffice to settle an essentially empirical question: What is the scientific explanation of the parallelogram law? But various familiar philosophical accounts of natural law not only

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Figure 1. (a) Forces *F* and *G* are represented as acting at a single point. (b) Their resultant *R* is represented by the diagonal of the parallelogram having *F*'s and *G*'s representations as adjacent sides.

make it difficult to see what the point of this dispute could have been, but also foreclose some options that were taken seriously in science.

The main point in dispute was whether the parallelogram of forces is explained by statics or by dynamics. If it is explained statically, then statics (the study of systems in equilibrium) is *autonomous*; it has its own laws. In particular, it is independent of dynamics (the study of systems in motion); the laws of statics are separate from and (in an important sense that I must elucidate) *transcend* the laws of dynamics. On the other hand, if the parallelogram of forces is explained dynamically, then "statics thus becomes a special case of dynamics, when the forces concerned happen to be in equilibrium" (Cox 1904, 68); the laws of statics become corollaries to the dynamical laws.

My concern is not to settle this controversy, which concerns a matter of contingent fact that is for empirical science to investigate.[1] My concern is *what it would be* for one or the other of these rival views to be correct. In particular, what would make it the case that the laws of statics transcend the laws of dynamics? The goal of this paper is to identify what the statical and dynamical interpretations are

[1] I discuss this point further at the start of section 5. By calling the question "empirical", I do not intend to suggest that some possible observation is logically consistent with a statical explanation of the parallelogram law but not with a dynamical explanation, or vice versa. Rather, I mean to suggest that questions like this one (i.e. questions concerning why some natural law holds) are the province of empirical science rather than a priori philosophy. Even if our most perspicacious application of "inference to the best explanation" in this case fails to yield a strong argument favoring either proposal as against its rival, I would not conclude that metaphysics should settle the matter. Rather, I would still insist that an adequate metaphysics should leave room for possible worlds where either proposal is true.

A useful comparison here is the question of whether the Lorentz transformation laws are explained dynamically (as Brown 2005 argues) or by the principle of relativity, various spacetime symmetries, and other facts about spacetime geometry (as many interpreters – and, I think, Einstein – believed). Perhaps different views regarding the order of explanatory priority here do not differ in their empirical predictions. Nevertheless, the issue concerns which scientific explanation is more plausible and so should be addressed not by metaphysical theories of natural law and scientific explanation, but empirically – that is, in the same way as we proceed in other cases (beloved by fans of underdetermination) where rival scientific explanations fit all of our observations (in the narrowest sense of "fit") but may not be equally well confirmed by them.

disagreeing about and to draw some metaphysical morals from their disagreement. Many accounts of natural law have been proposed, including Humean "Best System Accounts" (Lewis 1973, 1983, 1986, 1999), accounts involving nomic necessitation relations among universals (Armstrong 1978, 1983, 1997; Dretske 1977; Tooley 1977), and scientific essentialist accounts according to which laws reflect causal powers essential to possessing various properties (Ellis 2001, 2002; Bird 2007). None of these accounts (I shall argue) can make good sense of the dispute over the parallelogram law's explanation, since none of them nicely accommodates laws falling into *multiple* strata, with some laws having greater necessity than others and thereby transcending them. On any of these accounts, it is hard to see what it would be for the parallelogram of forces to belong to a higher stratum of natural law than the dynamical laws and so have no dynamical explanation. Moreover, scientific essentialism (I shall argue) has particular difficulty allowing for the possibility that the parallelogram of forces is not explained dynamically, since according to essentialism, a property's essence is exhausted by its causal role. According to essentialism, then, force's power to produce acceleration is precisely what should account for the way forces combine.[2]

If some accounts of natural law and scientific explanation cannot make good sense of this dispute over the parallelogram law's explanation, then it may be tempting to dismiss these failures as not at all serious considering the dispute's obscurity, apparently narrow scope, and somewhat antique character. However, I believe that this response would be mistaken. The dispute over the parallelogram law's explanation is not an isolated curiosity of merely mild antiquarian interest, but rather part of a much broader, longstanding controversy concerning the relation between statics and dynamics (cf. Duhem 1991) – and, even more broadly, concerning which physical facts (if any) are not ultimately to be explained dynamically (cf. Brown 2005). These disputes tie into a host of other general, live issues regarding non-causal scientific explanations, the roles played by spacetime symmetries and dimensional arguments in physics, explanatory levels and autonomy, and "constructive theories" and "bottom-up" explanations versus "theories of principle" and "top-down" explanations. (Einstein 1919; Salmon 1989, 182–183).

I will eventually suggest that *counterfactual conditionals* express what it would take for the parallelogram law to be grounded either in statics or in dynamics; different counterfactuals hold depending upon which sort of explana-

[2] Of course, there is a distinguished minority of natural philosophers who reject the standard interpretation of forces in classical physics as entities purportedly produced by various circumstances (such as fields or distant charges) and producing accelerations. I shall presuppose the standard interpretation since the scientists I discuss seem to do so in examining why the parallelogram law holds. This controversy's broader lessons apply whether or not forces, unreduced and uneliminated, should remain in our ontology.

tion the parallelogram law receives. In particular, for the laws of statics to transcend the laws of dynamics would be for the range of counterfactual circumstances under which the statical laws would still have held, corresponding to the necessity they possess, to extend beyond the range of counterfactual circumstances under which the dynamical laws would still have held, corresponding to the (weaker) variety of necessity they possess. This account, I will argue, makes good sense of what is at stake in the controversy concerning the parallelogram law's metaphysical status and proper explanation.[3]

2. *The dynamical explanation of the parallelogram of forces*

Those who endorse the dynamical explanation (e.g. Horsley 1743, 14; Rutherford 1748, 16–17; Bartlett 1855, 109; Thomson and Tait 1888, 244–245; Lock 1888, 15; Lodge 1890, 96; Loney 1891, 60; Pearson 1892/2007, 345–346; Routh 1896, 14; Cox 1904, 158) typically characterize it as the explanation that Isaac Newton (1687/1934, 15) gave, at least loosely, in deriving "Corollary 1" from his laws of motion. The dynamical explanation follows Newton's derivation in applying the second law (force = mass x acceleration) not only to the resultant force *R*, but also individually to each of the two component forces *F* and *G*, thereby grounding the parallelogram law in the two forces' independence. The dynamical explanation also follows Newton's argument in appealing to the "parallelogram of accelerations": that if a body simultaneously undergoes two accelerations represented in magnitude and direction by the adjacent sides of a parallelogram, then its net acceleration is represented by the parallelogram's diagonal. The dynamical explanation of the parallelogram of forces is that each of the two component forces is associated with an acceleration, those accelerations compose parallelogramwise, and the resultant acceleration reflects the resultant force. More rigorously:

> Suppose a point body, mass *M*, is acted upon by forces of magnitudes *F* and *G* in directions *f* and *g*, respectively, forming angle α. The forces are represented by line segments *MF* and *MG* in figure 2. By Newton's second law applied separately to the two forces, they cause accelerations of magnitudes *F*/*M* and *G*/*M* (in directions *f* and *g*), respectively, represented by line segments from *M* to points *F*′ and *G*′, respectively. Complete the parallelogram having those segments as adjacent sides. By the parallelogram of accelerations, the resultant acceleration is represented by the parallelogram's diagonal; its length is

[3] Although a counterfactual conditional's truth (or falsehood) cannot be observed, its truth can be confirmed (or disconfirmed) by empirical evidence. That is the way we came to know the many contingent facts we do know that are expressed by counterfactual conditionals. So in identifying counterfactual conditionals as expressing the facts in dispute regarding the parallelogram law, I place this controversy within the province of empirical science to investigate. (See the end of the paper.)

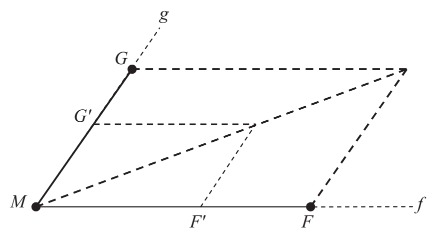

Figure 2. The dynamical explanation.

$$\sqrt{\left[(F/M)^2 + (G/M)^2 + 2(F/M)(G/M)\cos\alpha\right]}.$$

By Newton's second law[4], this resultant acceleration is associated with a resultant force directed along the diagonal with magnitude

$$M\sqrt{\left[(F/M)^2 + (G/M)^2 + 2(F/M)(G/M)\cos\alpha\right]},$$

which simplifies to

$$\sqrt{\left[F^2 + G^2 + 2FG\cos\alpha\right]}$$

– the length of the diagonal of the parallelogram having segments *MF* and *MG* as adjacent sides.

The standard objection to the dynamical derivation disputes not its cogency but its explanatory power – that is, whether (as its proponents contend) it contains "the most philosophical foundation for statics" (Cockle 1879, 12, characterizing Thomson and Tait 1888) or whether (as its critics charge) it is "unnatural and a defect in method" (Anon. 1829, 314). One common version of the objection is that the law determining the resultant of two forces is fundamentally nothing more than the law determining which forces combine to yield zero force, since forces *F* and *G* combine to yield force *R* if and only if *F*, *G*, and −*R* (a force equal and opposite to *R*) combine to yield zero force.[5] Furthermore, the law determining which forces combine to yield zero force (i.e. which forces are in equilibrium) is independent of what would happen in case of non-equilibrium. That is, the law is not based on the

[4] Newton's second law is *not* interpreted by advocates of the dynamical explanation as relating resultant acceleration and mass to the resultant force *as defined by the parallelogram of forces*. Doing so would defeat the point of giving a dynamical explanation of the parallelogram of forces. Newton's second law is interpreted as applicable to each component force separately, as well as to the resultant force *however forces turn out to compose*. The task is to show how the resultant force must therefore relate to the forces from which it results.

[5] This equivalence presupposes that equal and opposite forces have zero resultant and that the resultant of two forces is independent of which other forces may also be acting.

precise connection between force and motion. Therefore, notions such as motion, change, time, and mass (the proportionality constant relating unbalanced force to motion) should not enter into an explanation of the parallelogram law.

Of course, all of these notions figure prominently in the dynamical argument. It turns precisely on the relation between force and motion. But since there is no motion in an equilibrium case, and since the law of the composition of forces is fundamentally just the law specifying the conditions of equilibrium, the particular motion that would ensue under non-equilibrium is (the objection runs) explanatorily irrelevant. This is manifested by the fact that as we just saw, mass M is introduced in the dynamical argument but ultimately cancels out. According to the objection, M serves in the dynamical argument as an artificial device – like an arbitrary auxiliary line drawn to facilitate the proof of a geometric theorem, but having nothing to do with *why* the theorem holds.

Critics of the dynamical argument, such as James Challis, frequently praise William Whewell as having done "away with the illogical method of proving the parallelogram of forces by means of bodies in *motion*, which had previously been adopted in English works" (Challis 1875, 6). Whewell (1858, 225) contends that the parallelogram of forces is "independent of any observed laws of motion". He continues:

> The composition of motion . . . has constantly been confounded with the composition of force. But . . . it is quite necessary for us to keep the two subjects distinct. . . . The conditions of equilibrium of two forces on a lever, or of three forces at a point, can be established without any reference whatever to any motions which the forces might, under *other* circumstances, produce. . . . To prove such propositions by any other course, would be to support truth by extraneous and inconclusive reasons . . . (1858, 226).

Other critics, such as John Robison and W. E. Johnson, focus more explicitly on the dynamical argument's explanatory impotence:

> We cannot help being of the opinion, that a separate demonstration [of the law of composition of forces – separate, that is, from the composition of motions] is indispensably necessary. . . . [T]he composition of motions will not explain the composition of pressures, unless we take it for granted that the pressures are proportional to the velocities [i.e. that the force experienced by a body beginning at rest, multiplied by the duration over which the body experiences it, is proportional to the body's resulting velocity – a special case of Newton's second law]; but this is perhaps a gratuitous assumption (Robison 1822, 64; cf. Robison 1803, 599).
>
> The resultant . . . depends upon the components alone, and not upon the mass . . . of the body upon which the forces act. For this reason we find in many works upon mechanics the correctness of the parallelogram of forces demonstrated without reference to the mass . . . (Weisbach 1875, 178).
>
> [The dynamical argument] is open to . . . serious objections . . . [f]or it introduces kinetic ideas which are really nowhere again used in statics (Johnson 1889).
>
> [T]his cannot be considered perfectly satisfactory, as we are making the fundamental theorem of statics dependent upon a dynamical argument (A.G.G. 1890).

> [T]o make [the parallelogram of forces] . . . dependent on a theory of mass, as appears to be usual in modern text-books, somewhat grates upon one's sense of logical order (Macauley 1900, 403).
>
> Statics . . . , in itself, does not involve the ideas of motion and time. In it the idea of mass may also be entirely eliminated. Newton's proof of the parallelogram of forces has been objected to on the grounds that it requires the introduction of the fundamental conceptions of a much more complicated science than the one in which it is employed. [Other demonstrations] avoid this objectionable feature . . . (Moulton 1902, 5).

(Coming attractions: I shall unpack the sense in which dynamics is a "much more complicated science" than statics, according to critics of the dynamical explanation.)

That the parallelogram of forces is independent of Newton's second law is commonly put by saying that forces would still have combined in the same way even if force had been connected differently to motion – for example, even if the resultant force on a body had been proportional to the body's net velocity rather than its net acceleration. Daniel Bernoulli (1726/1982, 121), who gave a pioneering statical explanation of the parallelogram of forces, suggested that a wide range of alternatives to Newton's second law might have held, such as that the resultant force is proportional to the resultant acceleration's square root, or to its cube root, or to its square – but that even then, the parallelogram of forces would still have held.[6] On Bernoulli's view, the parallelogram law has greater necessity than Newton's second law, so the latter cannot be responsible for the former. Although Bernoulli maintained that the parallelogram of forces was geometrically necessary and thus knowable a priori, Bernoulli's objection to the dynamical explanation (namely, that the parallelogram of forces would still have held even if dynamics had not been governed by Newton's second law) continued to be taken seriously long after the parallelogram of forces had been recognized as not belonging to geometry and as knowable only empirically.[7] For example:

> [W]e may seek to arrive at the composition of pressures, independently of the second law of motion, by processes which are valid whether that law be a law of nature or not, and which would be valid even if we had not any conception of motion, and which, indeed, do not render it necessary to consider whether pressure does or does not tend to produce motion (Cockle 1879, 12–13).

[6] Of course, forces and accelerations cannot both combine parallelogramwise if any force is proportional to (say) the square of the acceleration for which it is responsible. Nevertheless, with forces and accelerations each combining parallelogramwise, the fundamental dynamical law could have been that force is proportional to acceleration squared as long as this law applies not to each component separately, but only to the *resultant* acceleration produced by the *resultant* force.

[7] De Morgan (1859) influentially argued that statical explanations of the parallelogram law are compatible with that law's being knowable only empirically. Others argued that the parallelogram law's explanation can be investigated independent of whether the law is knowable a priori or only empirically (e.g., Anon. 1850, 378).

> [I]f the velocities produced by these forces are not in the proportion of those intensities, but in the subduplicate ratio of them [that is, if the velocity resulting from the application of a given force for unit time to a given body, beginning at rest, is proportional not to the force's strength, as Newton's second law demands, but to the square root of the force's strength] . . . [then] this composition [that is, the composition of these forces] follows precisely the same rule as the composition of the forces which are measured by the velocities . . . but it does not appear that it can be held as demonstrated by the arguments employed in the case of motions . . . Accordingly, philosophers of the first eminence have turned their attention to this problem. It is by no means easy; being so nearly allied to first principles, that it must be difficult to find axioms of greater simplicity by which it may be proved (Robison 1822, 54–55).

Later I will use such counterlegals to specify what would make the parallelogram of forces have a statical explanation and so possess greater necessity than the second law of motion (albeit less necessity than mathematical truths).

The dynamical explanation's defenders frequently laud it as unifying the parallelogram of forces with the parallelogram of accelerations (and velocities), giving them a common origin: the parallelogram of displacements. For example, after characterizing Whewell's proposed statical explanation as "forced and unnatural", one "A.H." writes:

> With regard to mechanics itself, the extended and comprehensive view of this subject would embrace what are usually termed the parallelogram of forces and the composition of motion (or the "parallelogram of velocities," as it is sometimes called) in the same fundamental idea. The separation of these two things – which are in reality but one – is justly censured by Lagrange, as depriving them of their "evidence and simplicity" (A.H. 1848, 107).

Indeed, in a remark widely endorsed (e.g. by Cockle 1879, 13; Routh 1896, 18) as capturing an attractive feature of the dynamical explanation, Joseph Louis Lagrange (1813/1881, 349–350) says that the dynamical explanation "has the advantage of demonstrating clearly why the composition of forces necessarily follows the same laws as the one of velocity".

3. Duchayla's statical explanation

The statical explanation "usually given" (Browne 1883, 36; cf. Goodwin 1846, 272) in mid-nineteenth century textbooks (e.g. Earnshaw 1845, 9–10; Tate 1853, 72–74; Galbraith and Haughton 1860, 7; Goodeve 1874, 65–68; Todhunter 1878, 19–22; Church 1908, 4–6) originated with Charles Dominique Marie Blanquet Duchayla (1804). His explanation proceeds like a proof by mathematical induction. (For an interesting variant, see Dobbs 1901, 27–29.) I'll begin with Duchayla's explanation of the rule for the inductive step:

> If forces P and Q, acting together at a point, result in a force directed along the diagonal of the parallelogram representing the two forces, and likewise for forces P

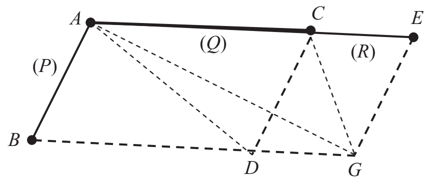

Figure 3. Duchayla's explanation, first part.

and R acting together at the same point, with R acting in Q's direction, then likewise for P together with the resultant of Q and R.

Here is Duchayla's explanation of this fact:

Let P be represented by segment AB (see figure 3). Grant that the resultant of Q and R is in their common direction and equal in magnitude to the sum of their magnitudes; let it be represented by segment AE, with Q represented by AC, so that segment CE is the proper length and direction to represent R except that R is actually applied at A rather than at C. (Draw the rest of figure 3 to complete the two parallelograms.) Nevertheless, when a force acts on a body, the result is the same whatever the point, rigidly connected to the body, at which it is applied, provided that the line through that point and the force's actual point of application lies along the force's direction.

(This constitutes the "principle of the transmissibility of force." Conversely, if a force's effect would be unaltered were its point of application changed to another point rigidly connected to its actual point of application, then the line through that other point and the force's actual point of application must lie along the force's direction.)

So although R is applied at A, its effect is the same if it is applied at C, since AC is in the force's direction. Continuing to treat the parallelograms in figure 3 as a rigid body, we can move the three forces' points of application to other points along the forces' lines of action without changing their resultant. We cannot move P's point of application directly to C, since AC does not lie along P's direction. But by hypothesis, the resultant of P and Q acts along diagonal AD, so the resultant can be applied at D. It can then be resolved into P and Q, now acting at D. Q's direction lies along DG, so Q can be transferred to G. P's direction lies along CD, so P can be transferred to C, where it meets R. By hypotheses, their resultant acts along diagonal CG, so it can be transferred to G, where it meets Q. By the converse of the force transmissibility principle, AG must lie along the line of action of the force resulting from P composed with the resultant of Q and R.

Thus Duchayla explains the rule for the inductive step. By symmetry (as I'll discuss further in the next section), the resultant of two equal forces acting at the same point is represented by an arrow from that point, in the same plane as the arrows representing the two forces, and bisecting the angle between them. This places us on the first "rung" of the "ladder" of mathematical induction; the inductive rule enables us to ascend the ladder to any height:

Start with a given force F, another force G with F's magnitude but in a different direction, and a third force H identical to G, all acting at the same point. Symmetry

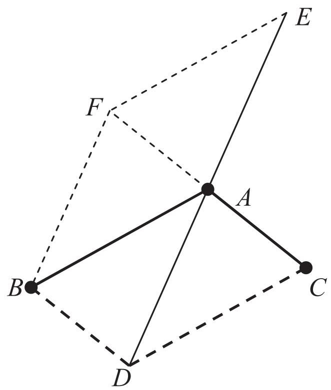

Figure 4. Duchayla's explanation, final part.

> gives the resultant of *F* and *G* as lying along the corresponding parallelogram's diagonal, and likewise for *F* and *H*. The inductive rule then shows that the resultant of *F* composed with a force of twice *F*'s magnitude – the resultant of *G* and *H* – points along the corresponding parallelogram's diagonal.
>
> Now take *F*, this force of twice *F*'s magnitude, and *H*. The inductive rule shows that the resultant of *F* together with a force of thrice *F*'s magnitude points along the corresponding parallelogram's diagonal – and so forth for *F* composed with a force having *n* times *F*'s magnitude (for any positive integer *n*).
>
> Now take a force in *F*'s direction but *n* times *F*'s magnitude, another force *G* with *F*'s magnitude but pointing in a different direction, and a third force *H* identical to *G*. By the inductive rule, the resultant of *nF* with a force of twice *F*'s magnitude points along the corresponding parallelogram's diagonal – and so forth for *nF* composed with *mF* (for any positive integers *n*, *m*), i.e. for any two commensurable forces.
>
> The explanation extends to the resultant of any two incommensurable forces since to any desired degree, there is a pair of commensurable forces approximating them to that degree.

So much for the resultant's direction. The final step of Duchayla's explanation shows why the parallelogram of forces also yields the resultant's magnitude:

> Let *AB* and *AC* (figure 4) represent two forces; their resultant (as we know) points in the direction of *AD* (the parallelogram's diagonal). Let *AE* extend *AD* in the opposite direction, its length representing the resultant force's magnitude. Now let's show that the parallelogram's diagonal represents the resultant's magnitude by showing *AD*'s length to equal *AE*'s. The forces represented by *AB*, *AC*, and *AE* balance. Construct parallelogram *AEFB* on *AB* and *AE*; we know that the resultant of the forces represented by *AB* and *AE* points along its diagonal *AF*. Hence *AF* must lie along the same straight line as *AC*. As opposite sides of parallelogram *ACDB*, *AC* and *BD* are parallel, and since *AF* and *AC* lie along the same line, *AF* is parallel to *BD*. As opposite sides of parallelogram *AEFB*, *AE* and *BF* are parallel, and since *AE* and *AD* lie along the same line, *AD* is parallel to *BF*. Since *AF-BD* and *AD-BF* are pairs of parallels, *AFBD* is a parallelogram. Its opposite sides *AD* and *BF* are equal in length.

> But as opposite sides of parallelogram *AEFB*, *AE* and *BF* are equal in length. So *AD* and *AE* (as both equal to *BF*) are equal.[8]

Some regard Duchayla's explanation as "very simple and beautiful" (Mitchell, Young, and Imray 1860, 47; cf. Earnshaw 1845, v; Pratt 1836, 7). Others, while admitting that the argument is sound, consider it:

> forced and unnatural . . . a considerable waste of time (Besant 1883, 581; cf. Lock 1888, 155);
> [t]he proof of our youth . . . now voted cumbrous and antiquated, and only retained as a searching test of logical power (A.G.G. 1890, 413);
> brainwasting . . . elaborate and painstaking, though benumbing (Heaviside 1893, 147); and (my favorite)
> certainly convincing . . . but . . . essentially artificial . . . *cunning* rather than honest argument (Goodwin 1849, 273).

Apart from name-calling, two specific deficiencies in Duchayla's explanation are cited. First, the principle of the transmissibility of force is "extraneous" (Macauley 1900, 403). As W. E. Johnson says,

> To base the fundamental principle of the equilibrium of a *particle* on the 'transmissibility of force', and thus to introduce the conception of a *rigid body*, is certainly the reverse of logical procedure (Johnson 1889, 153; cf. Anon. 1850, 378).

Just as the standard objection to the dynamical explanation was sometimes expressed in terms of a counterlegal ("The parallelogram of forces would still have held even if dynamics had not been governed by Newton's second law"), so likewise for the objection to Duchayla's explanation. In Samuel Earnshaw's words:

> [Duchayla's] method is inapplicable when the forces act upon a single [i.e. separate, unattached] particle of matter (as a particle of a fluid medium on the hypothesis of finite intervals), on account of its assuming the transmissibility of the forces to other points than that on which they act . . . [Duchayla's] method . . . can never be exclusively adopted in a treatise which professes to take a more philosophical view of the subject; for [here comes the counterlegal!], were the transmissibility of force *not* true in fact, the law of the composition of forces acting on a point would still be true; it is evident, therefore, that to make the truth of the former an essential step in the proof of the latter, is erroneous in principle (Earnshaw 1845, v).

This reasoning sheds further light on the relation between counterlegals and explanatory priority, which arose earlier in criticisms of the dynamical explanation and which will be central to section 6.

In contrast, fans of Duchayla's proof depict statics as concerned fundamentally with rigid extended bodies rather than material points. For example,

[8] Whewell's (1847, 30–32) explanation of the parallelogram law is similar, though it does not proceed inductively and, in place of the transmissibility principle, uses the principle that on any lever (a rigid rod, bent at a fulcrum), two forces tending to turn it oppositely balance exactly when they have equal "moments" about the fulcrum. (A force's "moment" equals its magnitude times the length of the line from the fulcrum to the line in the force's direction through the force's point of application.)

according to Challis (1869, 98), "Statics is restricted to the equilibrium of *rigid* bodies".

The second deficiency that critics identified in Duchayla's explanation is that it derives the parallelogram law's holding with regard to direction separately from its holding with regard to magnitude. Though it uses the former result to arrive at the latter, it requires additional steps to do so. As Harvey Goodwin says:

> [T]he extreme simplicity of this part of the proof [i.e. the extension to magnitude] shews how intimate the connexion must be between the two parts of the proposition, a connexion which I think we should not have been led to expect from anything occurring in the proof itself . . . [From the proof of the parallelogram law's holding with regard to direction] there is not a shadow of a hint that . . . the law will hold as respects magnitude: so that a very remarkable proposition is proved by a mere artifice without apparently the least reason in the nature of things why we should anticipate the result (Goodwin 1849, 273).

As Goodwin sees it, Duchayla's proof, while sound, mistakenly depicts as *coincidental* the fact that the parallelogram's diagonal represents the resultant in *both* direction and magnitude.

4. Poisson's statical explanation

After Duchayla's, the next most common statical explanation of the parallelogram law is Siméon Denis Poisson's (1811, 11–19; 1833/1842, 36–42).[9] (Among texts endorsing Poisson's explanation are Young 1834, 250–252; Barlow 1848, 10–11; and Price 1868, 19–21.) Instead of the transmissibility principle, the explanans includes symmetries, dimensional considerations, and the fact that two forces have a unique resultant determined entirely by their magnitudes and directions. (For example, their resultant is independent of what other forces are acting. Thus, if forces *A* and *B* have *C* as their resultant, then they still do so even if force *D* is also acting, so the resultant of *A*, *B*, and *D* is the resultant of *C* and *D*.)

First, Poisson explains the resultant of equal forces P_1 and P_2, each having magnitude *P*, acting in directions subtending angle $2x$. The explanans includes that the law governing the composition of forces treats all directions alike in that (roughly speaking) given the angle subtended by the two forces, their relation to their resultant is the same whatever the two forces' directions. That is, the composition law is rotationally symmetric: if we alter the direction of each component force by the same arbitrary angle, then their resultant alters by the same angle.[10] By

[9] Poisson develops a strategy pursued earlier by Foncenex and D'Alembert. Another strategy in the same spirit, but different in the details, was pursued by Daniel Bernoulli, Laplace, and Cauchy. Robison (1822, 57–64) presents a hybrid.

[10] Van Fraassen (1989, 268–270) briefly emphasizes symmetry's role in "dictating" the parallelogram law.

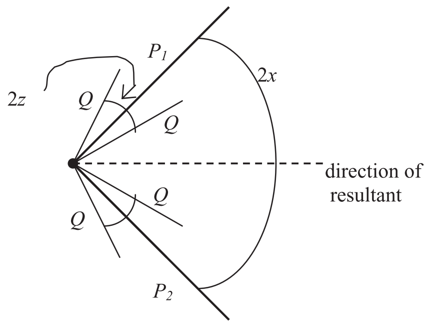

Figure 5. Poisson's explanation, first part.

this symmetry, the resultant must be coplanar with and bisect the angle between P_1 and P_2, and (since it reflects only the forces' magnitudes and directions) its magnitude R must be some function f of P and x alone. The explanans also includes that f is continuous (i.e. R various continuously with P and x) and the law is "dimensionally homogeneous": $R = f(P,x)$ holds in any system of units for R, P, and x. Then since angle is dimensionless, $f(P,x)$ must equal $P^{\alpha}g(x)$ for some dimensionless α and some dimensionless function g.[11] Since R and $P^{\alpha}g(x)$ must both have dimensions of force, and P has dimensions of force, it follows that $\alpha = 1$, and so $R = P\ g(x)$. Now to explain g:

Create another instance of the same problem (i.e. two equal forces acting in different directions): let P_1 be the resultant of two equal forces, each of some magnitude Q, subtending some angle $2z$ (see figure 5). Hence $P = Q\ g(z)$. Since $R = P\ g(x)$, it follows that $R = Q\ g(z)g(x)$. By the composition law's rotational symmetry, these two forces rotated through the same angle have P_2 as their resultant. The resultant of the four Q forces is the resultant of P_1 and P_2, with magnitude R.

The two inner Q forces in figure 5 create yet another instance of the same problem: their resultant bisects the angle $2(x - z)$ between them, and so points in the same direction as the resultant of P_1 and P_2, and has magnitude $Q\ g(x - z)$. Likewise, the resultant of the two outer Q forces is in that same direction, magnitude $Q\ g(x + z)$.

The explanans includes that if two forces point in the same direction, their resultant's magnitude is the arithmetic sum of their magnitudes. Therefore, as the

[11] Poisson, following Foncenex and D'Alembert, appeals explicitly to dimensional homogeneity but does not specify that this step presupposes f's continuity. For proof of the theorem behind this step, see any text on dimensional analysis, such as Bridgman (1931, 21–22). See also (Lange forthcoming-a) on scientific explanations that employ dimensional reasoning.

magnitude of the four Q forces' resultant, R equals the sum of $Q\ g(x-z)$ and $Q\ g(x+z)$. Since we found $R = Q\ g(z)g(x)$,

$$Q\,g(z)g(x) = Q\,g(x-z) + Q\,g(x+z), \text{ so}$$

$$g(z)g(x) = g(x-z) + g(x+z).$$

The solution to this functional equation is $g(z) = 2 \cos az$, for arbitrary a.[12] Therefore,

$$R = P\,g(x) = 2P\cos ax.$$

To determine a, let the explanans include that if two forces are equal and opposite, then they have zero resultant.[13] That is, $R = 0$ when $x = \pi/2$.

(Before continuing, note that although Poisson does not say so, this fact follows from the others that have already been included in the explanans: By rotational invariance, if equal and opposite forces are rotated 180° around an axis perpendicular to their common line of action, then the resultant is rotated 180°, but since the forces have merely swapped places, the resultant is unchanged, since the resultant depends only

[12] Poisson (1811, 14–16) solves the equation roughly as follows. (Poisson 1833/1842 takes a different approach; Perkins 1842 uses a third approach.) Let the explanans include that for all x, g is infinitely differentiable on some open interval containing x; that for all positive integers n, there are positive numbers r, M such that $|g^{(n)}(x+z)| \leq M$ for all $(x+z)$ in $\langle x-r, x+r\rangle$; and that likewise for $(x-z)$. Then expanding by Taylor series,

$$g(x+z) = \sum_{k=0}^{\infty} z^k g^{(k)}(x)/k!$$

$$g(x-z) = \sum_{k=0}^{\infty} (-z)^k g^{(k)}(x)/k!$$

Hence, $g(z)g(x) = g(x-z) + g(x+z)$ yields

$$g(z)g(x) = 2[g(x) + [g^{(2)}(x)]z^2/2! + [g^{(4)}(x)]z^4/4! + \ldots], \text{ so}$$

$$g(z) = 2[1 + [g^{(2)}(x)]z^2/2!\,g(x) + [g^{(4)}(x)]z^4/4!\,g(x) + \ldots].$$

In this expansion of $g(z)$, x must not appear, since $g(z)$ does not depend on x. So, for example, the $z^2/2!$ term's coefficient $[g^{(2)}(x)]\ /g(x)$ must be independent of x – and obviously it cannot depend on z, either. So it must be a constant b. Hence $g^{(2)}(x) = b\ g(x)$. Differentiating twice, $g^{(4)}(x) = b\ g^{(2)}(x) = b^2\ g(x)$, so the $z^4/4!$ term's coefficient $g^{(4)}(x)\ /\ g(x) = b^2$, and so forth for the other terms. Therefore,

$$g(z) = 2[1 + bz^2/2! + b^2 z^4/4! + b^3 z^6/6! + \ldots].$$

Letting $b = -a^2$,

$$g(z) = 2[1 - a^2 z^2/2! + a^4 z^4/4! - a^3 z^6/6! + \ldots].$$

The bracketed expression is the Taylor expansion of cos az.

[13] I simplify slightly this step of Poisson's argument, following Young (1834, 252).

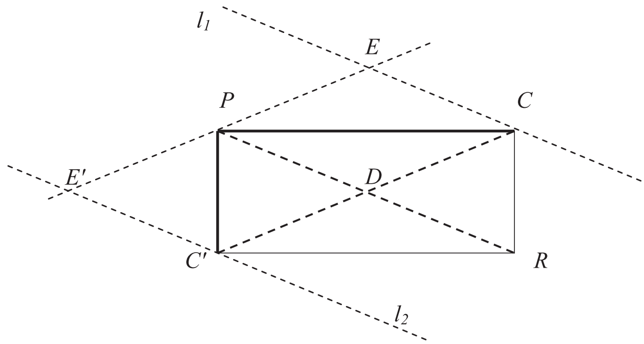

Figure 6. Poisson's explanation, second part.

on the forces' magnitudes and directions. The only resultant unchanged by being rotated 180° is a resultant of zero.)

By $R = 2P \cos ax$, $\cos a\pi/2 = 0$, so a must be an odd integer. If a is 3 or 5 or . . . , then $R = 0$ for $a \neq \pi/2$. This is disallowed by letting the explanans include that if two forces have zero resultant, then they are equal and opposite.

(This, too, follows from what has already been included in the explanans: If forces *A* and *B* have zero resultant, then the resultant of *A*, *B*, and some third force is the third force – but if the third force is equal and opposite to *B*, then by the previous result, it and *B* have zero resultant, so the three forces' resultant is *A*, which contradicts the resultant's uniqueness unless *A* and the third force are the same, i.e. unless *A* is equal and opposite to *B*.)[14]

Hence, $a = 1$, so $R = 2P \cos x$, which is the length of the diagonal of the rhombus with side *P*, angle $2x$.

This result for two equal forces can be extended to unequal, orthogonal forces:

Suppose they are applied at *P* (see figure 6) and represented by *PC* and *PC′*. Complete rectangle *CPC′R*. Let's use the above result to decompose each of these two forces into two equal forces: one along the rectangle's diagonal, the other cancelling its partner from the other force's decomposition. Draw diagonals meeting at *D*. Draw line l_1 (l_2) through *C* (*C′*) parallel to *PR*. Draw line through *P* parallel to *CC′*; let *E* (*E′*) be its point of intersection with l_1 (l_2). Since the diagonal *PR* bisects the diagonal *CC′*, $C'D = DC$, and so (since l_1 and l_2 are parallel to *PR*) $PE' = PE$. Treat *PE′* and *PE* as representing hypothetical equal and opposite forces acting at *P*, directed toward *E′* and *E* respectively; since they have no resultant, they can be joined with the *PC* and *PC′* forces without changing the resultant. *EPDC* (*E′PDC′*) is a rhombus, so (by the result just shown) *PC* (*PC′*) represents the resultant of the forces represented by *PE* (*PE′*) and *PD*. Consequently, the resultant of the forces represented by *PC* and *PC′* is the force represented by the resultant of *PD*, *PD* (again), *PE*, and *PE′*, i.e. is twice the force represented by *PD*, which is the force represented by the diagonal *PR*.

[14] The core of this argument appears in (Barlow 1848, 9).

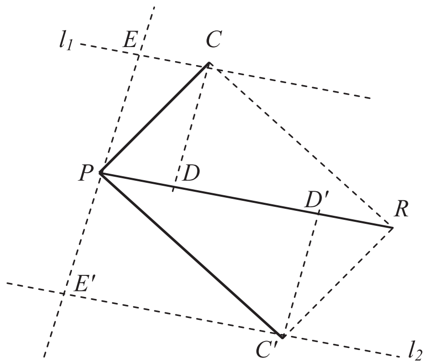

Figure 7. Poisson's explanation, third part.

Finally, a similar strategy extends this result to any two forces, completing the explanation of the parallelogram law:

> Suppose the two forces are applied at *P* (see figure 7) and represented by *PC* and *PC′*. Complete parallelogram *PCRC′*. Draw l_1 (l_2) through *C* (*C′*) parallel to diagonal *PR*. Draw another line through *P* perpendicular to *PR*; let *E* (*E′*) be its point of intersection with l_1 (l_2). Draw a line through *C* (*C′*) perpendicular to *PR*; let *D* (*D′*) be its point of intersection with *PR*. Triangles *CRD* and *C′PD′* are congruent. (This follows by AAS: Both are right triangles; as opposite sides of a parallelogram, *PC′* = *CR*; angles *D′C′P* and *DCR* are congruent since *C′D′* is parallel to *DC* and *C′P* is parallel to *RC*.) Therefore, *CD* = *C′D′*, so *PE* = *PE′*. Treat *PE* and *PE′* as representing hypothetical equal and opposite forces acting at *P*, directed toward *E* and *E′*; since they have no resultant, they can be joined with the *PC* and *PC′* forces without changing the resultant. *CEPD* (*C′E′PD′*) is a rectangle, so (by the result just shown) *PC* (*PC′*) represents the resultant of the forces represented by *PE* (*PE′*) and *PD* (*PD′*). Consequently, the resultant of the forces represented by *PC* and *PC′* is the force represented by the resultant of *PD*, *PD′*, *PE*, and *PE′*, and so is the resultant of the forces represented by *PD* and *PD′*. But since triangles *CRD* and *C′PD′* are congruent, *DR* = *PD′*, so the resultant of the forces represented by *PD* and *PD′* is the force represented by *PR*.

Thus the parallelogram law is explained.

An alternative to Poisson's procedure may clarify how the parallelogram law for two equal forces can explain the parallelogram law for any two forces.[15] Suppose *PA* and *PB* (figure 8) represent any two forces. We will now show how *PC* represents the sum of pairs of equal forces that approximate the original two forces to any desired degree. Let *O* be *PC*'s midpoint; draw *OD* perpendicular to *PC*. Triangles *POD* and *COD* are congruent (by SAS: *OD* is a shared side, the angles at *O* are right angles, *PO* and *CO* are equal by construction). Hence, *PD* and *DC* represent equal forces. Their resultant is represented by *PC* (by the result already

[15] Something like this procedure seems to be suggested by Barlow (1848, 10–11), but the suggestion was not taken up by any later writers (as far as I know).

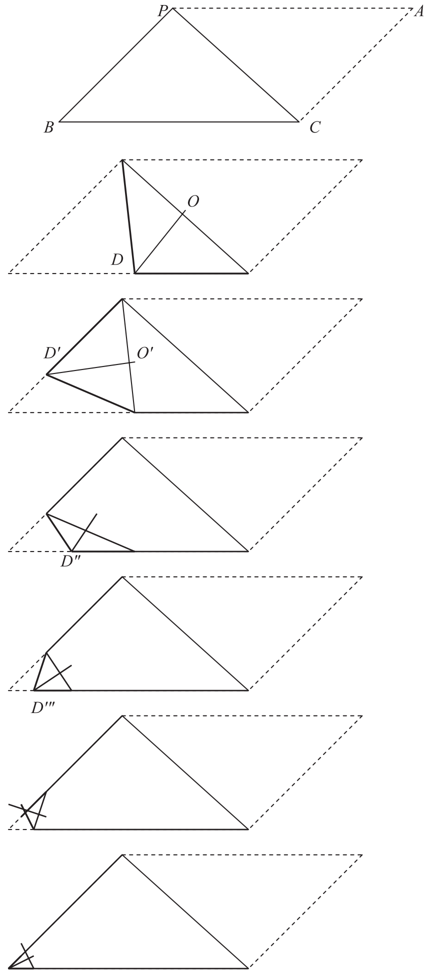

Figure 8. Poisson's explanation, fourth part.

shown for two equal forces). Now repeat this procedure: let O' be the midpoint of PD; draw $O'D'$ perpendicular to PD. The forces represented by PD' and $D'D$ are equal and compose to the force represented by PD, so those two forces with the force represented by DC result in the force represented by PC. By continuing this procedure as shown in figure 8, the successors of D, D' . . . approach arbitrarily closely to B, and the accumulated equal and opposite forces approach arbitrarily closely to the original two forces.

Unlike Duchayla's explanation, Poisson's treatment unites the parallelogram law's holding in magnitude with its holding in direction. Except in the explanation's first stage, where the resultant's direction follows immediately from symmetry, the resultant's direction and magnitude arise together throughout. Poisson's explanation disagrees with Duchayla's in depicting as no coincidence that the parallelogram's diagonal represents the resultant in both magnitude and direction.

Unlike the dynamical explanation, Poisson's explanation (in James Cockle's words) does "not render it necessary to consider whether pressure does or does not tend to produce motion" (Cockle 1879, 12–13). Indeed, nothing much about force in particular figures anywhere in Poisson's explanans, suggesting an objection expressed, though not endorsed, by Augustus De Morgan:

> [M]any have been puzzled by finding that the thing which, by its very definition, tends to produce motion, is reasoned on . . . under a compact that any introduction of the idea of motion would be out of place. The statical proofs . . . seem to be *all geometry and no physics* (1859, 299).

On the other hand, fans of Poisson's explanation see this feature as one of its distinctive contributions. It unifies the parallelogram of forces not only with the parallelograms of displacements, velocities, and accelerations, but also with the parallelograms of gravitational fields, bulk magnetizations, energy flux densities, water flux densities through soils, electric current densities through three-dimensional conductors,[16] sound and light flux densities through three-dimensional media, entropy fluxes, heat flows, temperature gradients, and so forth – all quantities that compose by vector addition. If all of these various parallelogram laws are explained by Poisson-style arguments, then it is no coincidence that such quantities, despite their physical diversity, all compose in the same way. Poisson's explanation shows why *any* physical quantity with certain basic features (given in Poisson's explanans) composes by a parallelogram law. As James Clerk Maxwell puts it:

> [T]he proof which Poisson gives of the "parallelogram of forces" is applicable to the composition of any quantities such that turning them end for end is equivalent to a reversal of their sign (1873, 10; cf. Anon. 1837, 43).

Even if there is no *common* reason why equal-and-opposites cancel for light flux density, water flux density, force, velocity, energy flux density, electric flux density through three-dimensional conductors, and so forth, there is (if Poisson-style explanations are correct) a basic similarity among them that explains why they all compose parallelogramwise.[17]

[16] However, electric current (as used in characterizing an electric circuit) is not a vector quantity – despite having a magnitude and being associated with a direction. After all, if two wires are connected to a given pole of a battery, one directed upward and one directed to the right, then it is not true that the net current flow is along the diagonal between them, since no wire points in that direction. The parallelogram law thus does not apply.

[17] That this basic similarity explains why these physically diverse quantities all compose parallelogramwise is like the fact that similarities in the fundamental equations of electrostatics and thermodynamics explain why various phenomenological laws in electrostatics and thermo-

5. Statical explanation under familiar accounts of natural law

We have seen some scientists argue that the parallelogram law is explained dynamically and others argue that it is explained statically. My aim is not to adjudicate this dispute, but rather to draw metaphysical morals from it. When (as in this case) there have been serious competing candidates for a given fact's scientific explanation, philosophical accounts of what is at stake should generally allow for the possibility (in some broad sense) that either candidate is correct. For example, there was once scientific controversy over whether the Copernican or Ptolemaic model of the heavens explains our observations of the night sky. A metaphysical account of natural law, cause, and scientific explanation should not have been enough to settle this controversy; any such account should permit Copernican possible worlds and Ptolemaic possible worlds (cf. Armstrong 1983, 26). Although there may be rare cases where a theory once taken seriously in science can safely be revealed by a philosophical account to have been a non-starter, I see no reason to think that philosophy alone should suffice to foreclose either candidate for explaining the parallelogram law.[18] Accordingly, let's now consider how the leading philosophical accounts of natural law could leave room for the possibility that the parallelogram law has a statical rather than a dynamical explanation.

According to David Lewis's "Best System Account", natural laws are distinguished from accidents by virtue of belonging to the best deductive system of truths. Some deductive systems of truths are stronger (i.e. more informative); others are simpler (e.g. by having fewer or simpler axioms). Typically, less strength brings greater simplicity. If "nature is kind" (Lewis 1999, 232–233), then one system has the optimal combination of strength and simplicity by any reasonable standard of strength, simplicity, and their trade-off, and this system is far ahead of any other. Its contingent members are the laws.[19]

dynamics are analogous. (One such similarity in the fundamental equations is that the rate of heat flow [the electric field] at a given point is proportional to the negation of the temperature [electrical potential] gradient there.) For more on the explanatory significance of what Maxwell termed "physical analogies", see Lange forthcoming-b.

Caution: Not all quantities characterized by a magnitude and a direction combine parallelogramwise. For instance, walking 3 meters east and then 4 m north is not equivalent to walking 4 m north and then 3 m east, if "equivalent" journeys must traverse the same ground, pass the same sights, and so forth.

[18] I do not intend to be presupposing a sharp distinction between metaphysics and science. Indeed, I have argued elsewhere that metaphysics is continuous with physics and that "philosophical" concerns have been integral to progress in physics (Lange 2002, 201). But I do insist that the dispute over the scientific explanation of the parallelogram law should not be capable of being settled merely by what it is to be a law of nature or a scientific explanation – any more than these philosophical accounts should suffice to preclude a geocentric cosmology.

[19] I have omitted some features of Lewis's account (such as the roles of chance and natural properties) that do not affect my discussion.

Advocates of a dynamical explanation see the parallelogram law and other laws of statics as corollaries to the dynamical laws – as all forming a single deductive system of laws. Advocates of a statical explanation acknowledge that the truth of the laws of statics can be deduced from the dynamical laws.[20] But these scientists regard the statical laws as *transcending* the dynamical laws, and so as forming their own deductive system of laws that omits the dynamical laws. The Best System Account could apparently accommodate advocates of a statical explanation by interpreting them as suggesting that these two systems are tied for best. (The statical system has less strength but greater simplicity than the more comprehensive system that also includes the dynamical laws.) Admittedly, Lewis (1999, 233) says that "if two very different systems are tied for best . . . there would be no very good deservers of the name of laws." But the statical and dynamical systems are not so very different; the statical system's members all belong to the dynamical system. Accordingly, we could perhaps amend the Best System Account to permit two strata of natural law just in case the two systems are tied for best and the members of one are all members of the other. Statics could then apparently qualify as autonomous – as having its own laws – and the existence of two "best systems" would not keep either system's members from deserving to qualify as laws. So amended, the Best System Account would apparently offer an interpretation of the point at issue between advocates of the statical and dynamical explanations: they disagree about whether the statical system is tied for best with the dynamical system.

However, this interpretation fails to capture what advocates of the statical explanation believe. In the more comprehensive system, dynamical laws constitute the axioms; statical laws are their corollaries. So in that system, statical laws depend on dynamical laws. The order of explanation in this system remains unchanged by the existence of another, equally good system where statical laws do not rest on dynamical laws. Advocates of the statical explanation do not believe that the parallelogram law is *also* explained dynamically. Rather, as we have seen, they believe that the statical picture "gives a better view of its true position", whereas the dynamical picture "grates upon one's sense of logical order" (Macauley 1900, 403). The Best System Account thus cannot easily be amended to reflect the parallelogram law's "true position" if it has a statical explanation.[21]

[20] Of course, that law *M* can be deduced from law *L* does not ensure that *M* is explained by *L*. This is one lesson commonly drawn from the notorious footnote 33 to Hempel and Oppenheim (1965, 273).

[21] Another strike against the Best System Account, as I have amended it, is that no scientists defended a statical explanation by arguing that the loss of strength from omission of the dynamical laws is balanced by the increase in simplicity.

As a functionalist, Lewis might have regarded laws of folk psychology as holding with broadly logical necessity, and hence with greater necessity than the laws of physics. (Thanks to a referee for pointing this out.) However, statics is not supposed to be a "special science"; it is part of fundamental physics. Moreover, on Lewis's view, the laws of statics are not broadly

This argument presupposes that an explanation of the parallelogram law consists of a deduction of the law from more fundamental laws – ultimately, on Lewis's account, from the axioms of (one of) the best system(s). But the argument is unaffected if this presupposition is weakened (in the spirit of Lewis 1986, 217–231) to acknowledge that an explanation of the parallelogram law might merely provide *some information* about how the law follows from the axioms – information that tells its recipient what he or she wanted to know about such a deduction.

If defenders of the statical explanation differed from defenders of the dynamical explanation in their interests and concerns, for example, then we might use that difference to account for their differences regarding the parallelogram law's explanation. A similar "pragmatic" account of the preference for a statical explanation might be available if they differed in the contrast classes they had in mind in posing their why-questions, or in the kinds of information they lacked, or in their beliefs about which facts are laws and which are accidents. But advocates of the statical explanation differed from advocates of the dynamical explanation in none of these respects. They did not pose their why-questions in different contexts. They both recognized the explanatory task as the deduction of the parallelogram law from more fundamental laws. That is why advocates of each view took themselves (as we have seen) to be genuinely *disagreeing* with advocates of the other view rather than as merely differing from them in their interests, priorities, or tastes regarding the style of explanation they prefer. Advocates of statical and dynamical explanations differed in their beliefs regarding the relation between statics and dynamics – roughly, in whether statics is just a special case of dynamics or "transcends" dynamics (in a sense that I will try to understand in the following section). This is a very old controversy (for some background, see Duhem 1991, e.g. pp. 184–186, 418–422); the dispute over the parallelogram law's explanation is just one manifestation of it. But it is this disagreement, rather than a more "pragmatic" difference, that ultimately accounts for the disagreement regarding the parallelogram law's explanation.

The same problem encountered by the Best System Account also arises on the accounts (proposed by David Armstrong, Fred Dretske, and Michael Tooley) depicting laws as irreducible, contingent relations of "nomic necessitation" among universals. Such an account can distinguish between "underived" and derived laws: the underived laws (nomic necessitation relations among universals) meta-

logical truths (and presumably this is so even if they transcend the dynamical laws). Thus, the prospect of the parallelogram law's having a statical explanation constitutes a more problematic case for Lewis's account than do the laws of folk psychology.

Of course, I have not demonstrated that it is impossible to reconcile Lewis's Best System Account with a statical explanation of the parallelogram law. At best, I have shown merely that one natural suggestion for doing so fails. After giving my own approach, I will be in a position to consider another way that one might try to stretch the Best System Account to permit a statical explanation (see note 28).

physically necessitate certain regularities among the particulars instantiating these universals, and these regularities, in turn, entail other regularities "that involve no new relations between universals" (Armstrong 1983, 145, cf. 173). These others are the derived laws. This distinction between underived and derived laws would appear to allow for an interpretation of the issue at stake regarding the parallelogram law. Advocates of the dynamical explanation seem to be holding that the parallelogram law and the other laws of statics are all derived laws, deriving from fundamental dynamical laws (so, as Cox says, "statics thus becomes a special case of dynamics"). Advocates of the statical explanation, on the other hand, seem to be holding that there are fundamental statical laws and that the parallelogram law is a derived law deriving from some of them. Thus the universals account seems to have identified the issue at stake.

However, once again, advocates of the statical explanation acknowledge that certain dynamical laws are underived laws and that they entail the parallelogram law's truth; the dynamical derivation is sound (but not explanatory). So on the picture of laws as relations among universals, there appears to be nothing that could possibly keep the fundamental dynamical laws from explaining the parallelogram law. Even if certain statical laws are underived, and so lie alongside the fundamental dynamical laws, the parallelogram law could at best be explained by a statical route *and* by a dynamical route. Once again, this is not the view defended by advocates of the statical explanation. Nothing in this picture accounts for why a statical explanation of the parallelogram law "gives a better view of its true position".[22]

This view of the parallelogram law is also difficult to accommodate within scientific essentialism (as defended by Brian Ellis and Alexander Bird), according to which laws reflect the causal roles essential to various properties and so are metaphysically necessary. For example, according to scientific essentialism, part of the essence of being electrically charged is having the power to exert (and the liability to feel) forces in accordance with Coulomb's law. But if all of the laws are metaphysically necessary, what could it be for the laws of statics to *transcend* – to be *more* necessary than – the laws of dynamics? To put all natural laws on a modal par with metaphysical necessities such as "Red is a color" (or, according to some philosophers, "Water is H_2O") is to put them all on a modal par with one another, which is incorrect if the laws of statics have greater necessity than the laws of dynamics.

There is a second reason why essentialism precludes the parallelogram law from having a statical explanation and thus takes sides in a controversy that should be decided by science rather than metaphysics. Essentialism takes causal powers

[22] An account of laws as metaphysically primitive (as in Maudlin 2007) has no resources in terms of which to elaborate what makes the parallelogram law, though following from dynamical laws, explained by statical rather than dynamical laws.

as fundamental and elaborates them in terms of the way in which something possessing a given power would respond to certain stimuli under certain conditions. For example, the property of being a point body, electrically charged to 1 statcoulomb, is elaborated in terms of subjunctive conditionals such as "If the body were exposed to an electric field at the body's location of 300,000 volts per meter, then the body would feel 10 dynes of electric force". This conditional's antecedent specifies a stimulus: an initial condition, logically consistent with the laws, that activates the causal power. Now as we have seen, physicists defending the statical explanation of the parallelogram law elaborate the autonomy of statics in terms of conditionals such as "Had forces and motions not accorded with Newton's second law, then the composition of forces would still have accorded with the parallelogram law". This counterfactual is a counterlegal; its antecedent posits a violation of the laws. Essentialists must interpret its antecedent as positing a metaphysical impossibility. It certainly does not posit a physically possible initial condition that engages a causal power; it is not the antecedent of any of the stimulus-response conditionals elaborating the causal powers that essentialists believe fundamental. Hence, those causal powers cannot underlie the parallelogram law's capacity to withstand changes to the dynamical laws. There is nothing in the essentialist picture to make the parallelogram law so resilient.

Here is another way to put this point. According to essentialism, laws are epiphenomenal; causal powers do all of the work that laws have traditionally been seen as doing. For example, causal powers figure in all scientific explanations (whereas laws, which describe regularities holding because of those powers, are explanatorily idle):

> Essentialists seek to expose the underlying causes of things, and to explain why things are as they are, or behave as they do, by reference to these underlying causal factors. Consequently, explanations of the sort that essentialists are seeking must always have two parts. They must contain hypotheses about the underlying structures or causal powers of things, and hypotheses about how things having these structures and powers must behave in the specific circumstances in which they exist (Ellis 2002, 159–160).

A dynamical explanation of the parallelogram law fits this model well. Essentialists could characterize a force as essentially a power to cause masses to accelerate in accordance with Newton's second law.[23] The dynamical explanation uses force's causal role to explain why it composes parallelogramwise.

[23] Of course, a *component* force must be the power to cause a mass to undergo a *component* acceleration. The parallelogram law for the composition of accelerations will, in turn, have to be explained by what accelerations are – and, ultimately, by what displacements are.

Some essentialists (e.g. Ellis 1990, 70) instead regard forces as a species of causal relation, not as causes themselves. (Recall note 2.) However, since this view is independent of essentialism, and since the moral I shall draw applies beyond forces to other vector physical quantities that all essentialists regard as causal actors, I shall construe essentialism as treating forces as causal actors.

However, Poisson's statical explanation of the parallelogram law does not fit this model. It does not exploit the causal power essential to force. Indeed, defenders of the statical explanation see its explanatory strength as deriving precisely from the fact that it *does not* exploit force's causal role. The statical explanation identifies certain features common to forces, electric currents, sound, light, heat, and many other, physically diverse quantities. According to the statical explanation, these quantities all compose parallelogramwise because of these common features (rather than because of their respective causal powers). Essentialism cannot recognize the statical argument as deriving its capacity to explain from its *failure* to exploit the essence of force.

Likewise, essentialism leaves no room for Olinthus Gregory's critique of the dynamical explanation:

> It may be proper to remark here that the Composition and Resolution of *forces*, and the similar Composition and Resolution of *motions*, are completely distinct objects of enquiry. . . . Some authors have inferred from their demonstrations of the latter problem, the truth of the former: but this cannot well be admissible, because wherever statical equilibrium obtains there can be no motion, and of course the principle on which the inference is grounded [namely, Newton's second law] is foreign to the nature of the thing to be proved (1826, 14–15).

Essentialism cannot accommodate Gregory's claim that Newton's second law is "foreign to the nature of the thing to be proved" (or Whewell's claim that it is "extraneous", as quoted earlier). On the contrary, according to essentialism, force's power to cause acceleration captures force's essence and is therefore precisely what *must* explain the parallelogram law. Essentialism cannot recognize force's causal role as "out of place" (as De Morgan said) in the explanation of the parallelogram law.[24]

[24] Of course, essentialism could allow force's causal role to explain the various symmetries and other facts figuring in Poisson's explanans, and then allow Poisson's explanation to proceed from there. But advocates of the statical explanation see the parallelogram law as deriving from a *deeper* source than force's causal role – so that the parallelogram law would still have held, even if that role had been different. Essentialism cannot embrace this view; according to essentialism, there is nowhere deeper than force's essence. Advocates of the statical explanation cannot believe that the facts figuring in Poisson's explanans hold only by virtue of force's causal role, since advocates of the statical explanation believe that those facts would still have held, had force's causal role been different. (Cf. Earnshaw 1845, v, quoted earlier.)

Not every explanation that might reasonably be termed "non-causal" poses this problem for essentialism. For example, "the explanation of a general law by deductive subsumption under theoretical principles is clearly not an explanation by causes" (Hempel 1965, 352), but essentialism has no difficulty in understanding how the causal powers behind Newton's laws of motion and gravitation give rise to the regularity associated with, say, the law relating the length and period of a classical pendulum. Some other "non-causal" explanations abstract away from the particular causes to exploit general features they possess. Peter Lipton gives a nice example:

> There also appear to be physical explanations that are non-causal. Suppose that a bunch of sticks are thrown into the air with a lot of spin so that they twirl and tumble

The same moral applies to Poisson-style explanations of why electric current densities through three-dimensional conductors, sound and heat flux densities through three-dimensional media, and so forth compose parallelogramwise: none of these explanations invokes what essentialism considers the essences of these qualities. Hence, essentialism cannot allow even for the *possibility* that Poisson-style arguments are explanatory and the causal powers associated with these various physical quantities fail to explain why they compose parallelogramwise.

6. *My account of what is at stake*

The account of natural laws that I have elaborated elsewhere (Lange 2005, 2006, 2007, 2009) can nicely capture what it would take for the laws of statics to *transcend* the dynamical laws by possessing a stronger variety of necessity than they do – and thus what it would be for the parallelogram law to have a statical rather than a dynamical explanation. The key to this account will be the subjunctive facts (expressed by counterlegals) used by advocates of the statical explanation to express the parallelogram law's independence from dynamics, such as the fact that forces would still have composed in the same way even if force had stood in a different relation to motion.

> as they fall. We freeze the scene as the sticks are in free fall and find that appreciably more of them are near the horizontal than near the vertical orientation. Why is this? The reason is that there are more ways for a stick to be near the horizontal than near the vertical. To see this, consider a single stick with a fixed midpoint position. There are many ways this stick could be horizontal (spin it around in the horizontal plane), but only two ways it could be vertical (up or down). This asymmetry remains for positions near horizontal and vertical, as you can see if you think about the full shell traced out by the stick as it takes all possible orientations. This is a beautiful explanation for the physical distribution of the sticks, but what is doing the explaining are broadly geometrical facts that cannot be causes (Lipton 2004, 9–10)

But what is doing the explaining here are also various abstract features of the causal powers at work, such as that the air molecules colliding with the stick are as likely to push it in one direction as in another and that the causal power of these forces to produce acceleration will introduce no asymmetry into the motion that was not already present in the forces. So essentialism can accommodate this explanation (if essentialism can manage to construe features of space, such as the feature Lipton identifies, "as the causal powers of some thing"). In contrast, abstract features of the relevant causal powers do not figure in the Poisson-style statical explanation of the parallelogram law. Even abstract features of force's power to produce acceleration make no appearance in the explanation. Whereas Lipton's explanation results from taking the actual multitude of petty causal influences on the tumbling sticks and abstracting from them, retaining only those features they possess that intuitively "make a difference" to the fact being explained (Strevens 2008), the parallelogram law's statical explanation cannot be generated by a similar process of abstraction from the dynamical explanation. Whereas Lipton's explanation exploits key features of the dynamical law and the forces on the sticks, the statical explanation involves nothing dynamical at all.

Let's start by considering the facts that we are trying to partition into laws and accidents – ultimately, into facts with various grades of necessity. These facts include that my pocket now holds an emerald, that all emeralds are green, etc. – but not that *it is a law* that all emeralds are green. That is, let's start with the "sub-nomic" facts: the facts that hold, in any possible world where they do hold, not in virtue of which facts are laws there and which are not. Many philosophers (e.g. Goodman 1983) have suggested that the sub-nomic facts that are laws would still have held under any sub-nomic counterfactual antecedent that is logically consistent with all of the sub-nomic facts that are laws. Trivially, no sub-nomic fact that is an accident is preserved under all of these antecedents.

Of course, this suggestion takes all of the logical truths, conceptual truths, mathematical truths, metaphysical truths, and so forth – the "broadly logical truths" – as included by courtesy among the natural laws, since they have at least the same perseverance under counterfactual antecedents as mere laws do. The suggestion, more fully, is:

> It is a law that *m* (where *m* is sub-nomic) if and only if in all conversational contexts, it is true that had *p* been true, *m* would still have been true [that is, $p \;\square\!\!\rightarrow m$], for any sub-nomic *p* that is logically consistent with all of the sub-nomic claims *n* (taken together) where it is a law that *n*.

This reference to the conditional's being true in *all conversational contexts* is required because the truth-values of counterfactual conditionals are notoriously context-sensitive.

This suggestion for distinguishing laws from accidents has an obvious problem: the laws appear on both sides of the "if and only if." The laws are picked out by their invariance under a range of antecedents that is, in turn, picked out by the laws. We have not only a vicious circularity in the analysis of lawhood, but also a privilege arbitrarily accorded to the laws: of designating the relevant range of antecedents. Invariance over this range makes the laws special only if there is already something special about this range – and hence about the laws.

But this problem can be avoided. The suggestion was roughly that the laws form a set of truths that would still have held under every antecedent with which the set is logically consistent. In contrast, take the set containing exactly the logical consequences of the accident that all gold cubes are smaller than a cubic meter. This set's members are *not* all preserved under every antecedent that is logically consistent with them all. For instance, had Bill Gates wanted a gold cube exceeding a cubic meter to be constructed, I dare say such a cube would have existed – yet that Bill Gates wants such a cube constructed is logically consistent with all gold cubes being smaller than a cubic meter.

That is the idea behind the definition of "sub-nomic stability". Consider a non-empty set Γ of sub-nomic truths containing every sub-nomic logical consequence of its members. Here is the definition:

> Γ possesses *sub-nomic stability* if and only if for each member m of Γ and for any sub-nomic claim p where $\Gamma \cup \{p\}$ is logically consistent (and in every conversational context), it is not the case that had p held, then m's negation might have held [that is, $\sim (p \Diamond\!\!\rightarrow \sim m)$, which entails that $p \Box\!\!\rightarrow m$].

Sub-nomic stability avoids privileging the range of counterfactual antecedents that is logically consistent with the laws.

The set Λ containing exactly the sub-nomic truths that are laws is stable, whereas the set spanned by the gold-cubes accident is unstable. Let's look at another example. Take the accident g that whenever a certain car is on a dry flat road, its acceleration is given by a certain function of how far its gas pedal is being depressed. Had the gas pedal on a certain occasion been depressed a bit farther, then g would still have held. Can a stable set include g? The set must also include the fact that the car has a 4-cylinder engine, since had the engine used 6 cylinders, g might not still have held. (Once the set includes the fact that the car has a 4-cylinder engine, the antecedent that the engine has 6 cylinders is logically *inconsistent* with the set, and so to be stable, the set does not have to be preserved under that antecedent.) But since the set includes a description of the car's engine, its stability also requires that it include a description of the engine factory, since had that factory been different, the engine might have been different. Had the price of steel been different, the engine might have been different. And so on – this ripple effect propagates endlessly. Take the antecedent: had either g been false or there been a gold cube larger than a cubic mile. Is g preserved? In every context? Certainly not. Therefore, to possess sub-nomic stability, a set that includes g must also include the fact that all gold cubes are smaller than a cubic mile (making the set logically inconsistent with the antecedent, so to be stable, the set does not have to be preserved under that antecedent). Since a stable set that includes g must include even the fact about gold cubes, I conclude that the only set containing g that might be stable is the set of *all* sub-nomic truths.[25]

I conclude that *no* nonmaximal set of sub-nomic truths that contains an accident possesses sub-nomic stability. Stability *is* possessed by Λ. Are any other nonmaximal sets stable? The sub-nomic broadly logical truths form a stable set

[25] According to many proposed logics of counterfactuals, $p \Box\!\!\rightarrow q$ is true trivially whenever $p\&q$ is true (a principle known as "Centering"). If Centering is correct, then each member of the set of all sub-nomic truths is trivially preserved under every sub-nomic supposition p that is true. Of course, there are no sub-nomic suppositions p that are false and logically consistent with the set. (If p is a false sub-nomic supposition, then $\sim p$ is a member of the set.) Hence, if Centering holds, then the set of all sub-nomic truths trivially possesses sub-nomic stability. Accordingly, I will argue that Λ is the largest *nonmaximal* set that is sub-nomically stable.

However, I am not convinced that Centering holds generally (for example, in a world governed by irreducibly statistical laws). Therefore, I will not endorse the stability of the set of all sub-nomic truths.

since they would still have held under any broadly logical possibility. Let's prove that for any two sub-nomically stable sets, one must be a proper subset of the other. The strategy is to consider an antecedent pitting the invariance of the two sets against each other:

1. Suppose (for *reductio*) that Γ and Σ are sub-nomically stable, *t* is a member of Γ but not of Σ, and *s* is a member of Σ but not of Γ.
2. Then (~*s* or ~*t*) is logically consistent with Γ.
3. Since Γ is sub-nomically stable, every member of Γ would still have been true, had (~*s* or ~*t*) been the case.
4. In particular, *t* would still have been true, had (~*s* or ~*t*) been the case. That is, (~*s* or ~*t*) □→ *t*.
5. So *t* & (~*s* or ~*t*) would have held, had (~*s* or ~*t*). Hence, (~*s* or ~*t*) □→ ~*s*.
6. Since (~*s* or ~*t*) is logically consistent with Σ, and Σ is sub-nomically stable, no member of Σ would have been false had (~*s* or ~*t*) been the case.
7. In particular, *s* would not have been false, had (~*s* or ~*t*) been the case. That is, ~((~*s* or ~*t*) □→ ~*s*).
8. Contradiction from 5 and 7.

We were asking about nonmaximal sub-nomically stable sets besides Λ. Since no nonmaximal *superset* of Λ is sub-nomically stable (since it would have to contain accidents), we must look for sub-nomically stable sets among Λ's proper subsets. Many of them are clearly unstable. For instance, take Coulomb's law (the law governing electrostatic forces). Suppose we restrict it to times *after* today and take the set containing exactly this restricted law and its broadly logical, sub-nomic consequences. This set is unstable since it is false that the restricted law would still have held, had Coulomb's law been violated sometime *before* today.

However, some of Λ's proper subsets *are* plausibly sub-nomically stable. For instance, take a set containing exactly the sub-nomic broadly logical consequences of the fundamental law of dynamics (in Newtonian physics: Newton's second law of motion), the conservation laws, the parallelogram of forces, etc. – without any of the force laws (e.g. Coulomb's law, Newton's gravitational-force law, etc.) It is widely thought that this set's members would still have held, had some force law been different. For example, to ascertain how bodies would have behaved had gravity been an inverse-cube force, we use the fundamental law of dynamics. Paul Ehrenfest (1917) thereby famously showed that had gravity been an inverse-cube force, the planets would eventually have collided with the sun or escaped from the sun's gravity. Scientists regard the fundamental dynamical law as possessing a stronger variety of necessity than the gravitational force law does. Likewise, scientists do not believe that the only reason that energy is conserved is because one kind of fundamental interaction conserves energy, another kind independently does so, too, and likewise for each other kind of fundamental interaction. If scientists believed energy conservation to be just such a coincidence, then they could not consistently believe (as they do[26]) that were there additional kinds of fundamental interaction (perhaps taking place only under exotic, unfamiliar conditions), then they too would conserve energy. This subjunctive conditional is not merely *unlikely* to be true if energy conservation is coincidental. It is *false*.

The various strata of natural law thus form a hierarchy of sub-nomically stable sets (depicted in Figure 9; ignore the "*" for the moment):

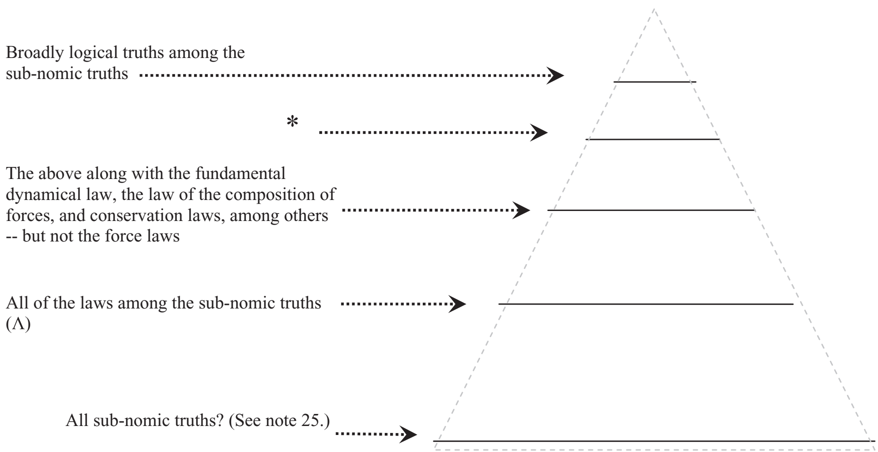

Figure 9. Some (though perhaps not all) plausibly sub-nomically stable sets.

For every grade of necessity, the truths possessing it form a nonmaximal stable set, and for each nonmaximal stable set, there is a variety of necessity that is possessed by all and only its members. There are good pretheoretic reasons to identify stability with necessity. A stable set has *maximal* staying power under antecedents: its members would all still have held under every sub-nomic supposition under which they *could* without contradiction all still have held. They are *collectively* as resilient under sub-nomic suppositions as they could *collectively* be. This sounds like necessity to me.

On this picture, there are many species of natural necessity – many strata of laws. A stable proper subset of Λ is associated with a stronger variety of necessity than Λ. That is, the range of antecedents under which the proper subset's members

[26] See Bergmann 1962, 144; Feynman 1967, 59, 76, 83, 94; Planck quoted in Pais 1986, 107–108.

are all preserved, in connection with its stability, is wider than the range of antecedents under which Λ's members are all preserved, in connection with Λ's stability. The conservation laws thereby *transcend* the force laws.

I suggest that advocates of the statical explanation believe that the laws of statics, including the parallelogram law, transcend even the fundamental dynamical law and conservation laws. They (together with the broadly logical truths) form a sub-nomically stable set even higher in the pyramid (though below the broadly logical truths) – at the level labeled with "*" in Figure 9. Thus, they possess a stronger variety of necessity than the dynamical laws do. Although a derivation of the parallelogram law from the dynamical laws is sound, this derivation cannot explain the parallelogram law because the premises lack the stronger variety of necessity that the parallelogram law possesses. The parallelogram law cannot rest on the dynamical laws because the dynamical laws, lacking the necessity possessed by the parallelogram law, cannot be responsible for the parallelogram law's possessing it. In connection with its membership in an exclusive stable set, the parallelogram law is preserved under certain counterfactual antecedents, and its preservation there cannot be explained by its following from the dynamical laws since the dynamical laws are not preserved there. If advocates of a statical explanation are correct, then although the dynamical laws entail the parallelogram law's truth, their necessity is too weak to entail its characteristic necessity.[27]

In elaborating what it would be for the statical laws to be independent of the dynamical laws, I have appealed to precisely the sort of counterlegals invoked by advocates of the statical explanation. Recall also that in trying to amend Lewis's account to accommodate various levels of natural law, I had to *stipulate* that even if there are many systems tied for best, there are truths that nevertheless deserve to count as laws just in case for any two of the best systems, the members of one are all members of the other. My account requires no such stipulation, since as we have seen, it falls naturally out of my account that for any two stable sets, one must be a proper subset of the other. Thus, my approach not only teases the various grades of natural necessity apart, but also explains *why* there is a natural ordering among them.[28]

[27] It is not the case that advocates of the statical explanation (as I understand them) regard the dynamical laws as able to explain why the parallelogram law is *true*, and even why it is *a law* (since it follows exclusively from laws), but as unable to explain why it has its characteristic necessity (which is stronger than any necessity possessed by the dynamical laws). Rather, the parallelogram law is true (and has the weaker flavor of necessity characteristic of the dynamical laws) because it is a law (with the stronger flavor of necessity). As I noted in section 5, advocates of the statical explanation do not believe that the parallelogram law's truth is *also* explained dynamically.

[28] We can now entertain another proposal (suggested by John Roberts) for reconciling the Best System Account with the possibility that the parallelogram law is to be explained statically. Stipulate that if two systems are tied for best (where one set's members are a proper

Of course, I am not arguing that advocates of the statical explanation are correct. That is an empirical question. I have merely tried to identify the contingent facts responsible for determining the order of explanatory priority here – that is, the facts over which advocates of the statical and dynamical explanations of the parallelogram law are ultimately disagreeing. I have identified these as various subjunctive facts, expressed by certain counterfactual (indeed, counterlegal) conditionals.

These subjunctive facts are to be ascertained empirically. Counterfactuals, on my view, are confirmed by evidence no differently from other claims:

> Our past observations of emeralds confirm not only that all of the actual emeralds lying forever undiscovered in some far-off land are green, but also that had there been an emerald in my pocket right now, then my pocket would have contained something green. (It is not self-evident which of these predictions is more "remote" from our observations.) When we confirm that my pocket would contain something green were there emeralds in it, that confirmation is unaffected by whatever evidence we may have regarding whether there actually are any emeralds in my pocket. So in confirming that my pocket would contain something green were there emeralds in it, we may be confirming both a prediction about the actual world and a counterfactual conditional. Facts about what *would have been* are confirmed right along with facts about what actually *is* (Lange 2009, 11).

The same applies to the confirmation of counterlegals. For instance, scientists typically regard the space-time symmetries of known force laws as confirming that the same symmetry principles hold of whatever unknown laws govern as yet undiscovered kinds of forces (see Lange 2007, 2009). As in the emerald example, this confirmation fails to discriminate between actual unexamined and counterfactual cases; the evidence confirms that had the laws of nature been different so that

subset of the other's), then for any fact that is in both, the smaller set "gives a better view of its true position"; such a fact is explained not by the larger set's axioms, but by the axioms of the smaller set. Advocates of the statical explanation could then be disagreeing with advocates of the dynamical explanation over whether or not the statical and dynamical systems are in fact tied for best.

But why does a smaller best set's deduction take precedence for the purposes of explanation? Because (the proposal continues) both sets are sub-nomically stable, their stability is associated with their lawhood, and a deduction from the larger set's axioms cannot explain why a member of the smaller set would still have held under counterfactual antecedents logically consistent with the smaller set but not with the larger. Its invariance under these antecedents is associated with its characteristic necessity.

Of course, this last is very much like the account that I propose (as tying in nicely with the arguments actually given by advocates of the statical explanation). But the Best System Account cannot be brought this close to my account without losing its Best System character. For instance, the laws can no longer straightforwardly supervene on the sub-nomic facts if the laws are to be sub-nomically stable, since stability requires that the laws remain invariant despite radical changes to the sub-nomic facts – sufficiently radical to alter which facts form the system with the best combination of simplicity and strength. (See Lange 2009.) For example, had there been nothing except a single electron moving uniformly forever, then the best deductive system would have been quite different.

there had been an additional or different kind of force, its laws would have exhibited the same symmetry. Evidence bears on the forces there would have been for the same reason as it bears on the unknown forces there are.

Of course, without some background opinions regarding the relation between the way things would have been and the way things are observed to be, we cannot use our empirical evidence to confirm counterfactual conditionals. But this is no objection to my view that counterfactuals are confirmed empirically: the analogous point holds for the confirmation of claims regarding actual unexamined cases. Indeed, I have argued (Lange 2004) that even when our accepted background theory is highly impoverished, we cannot make any observations at all unless we possess the sorts of background opinions that could underwrite the confirmatory power of those observations – including their power to confirm counterfactual conditionals. But as van Fraassen (1980, 19) nicely puts it, "we cannot settle the major questions of epistemology *en passant* in philosophy of science"!*

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