An evaluation of sensory noise in the human visual system

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Abstract. It is assumed that the activity of a visual channel may be represented as $V(t) = g(t) + \xi(t)$, where $g(t)$ is the deterministic response of the channel due to the presentation of a stimulus and $\xi(t)$ is the trajectory of a wide-sense stationary Gauss process. The stimulus is detected if the event $\{V(t) > S$ for at least one $t \in [0, T]\}$ occurs. Two approximations for the probability of this event are proposed, and it is demonstrated how they may be employed to estimate (i) the value of the second spectral moment λ_2 of the noise process ξ_t, where λ_2 reflects the speed of the fluctuations of the trajectories $\xi(t)$, and (ii) the value of the internal threshold S. The commonly made assumption of peak – detection is shown to serve as a very good first approximation in particular if the channel is of transient type or – in case of detection by a channel of sustained type – if the stimulus durations are not too long.

1 Introduction

In this paper, some estimates of characteristics of the noise within the visual system will be derived from psychophysical detection data. At the same time we arrive at an evaluation of the frequently made assumption of *peak-detection* as compared to that of detection by *temporal probability summation* (TPS). It is assumed (i) that the sensory activity may be represented as a nonstationary stochastic process, (ii) that detection is a level-crossing process.

We present two approximations for the probability of such level-crossings. They will be employed to estimate (i) the rate of fluctuation of the noise and (ii) the value of an internal threshold.

It will be shown that the classical postulate of peak-detection may be considered as a special case of a level-crossing process. Peak-detection allows a very direct interpretation of detection thresholds, provided the system is linear: the reciprocal of the threshold contrast or intensity is proportional to the maximum value of

the (mean) response. Clearly, such estimates of the maximum value will be distorted when the assumptions of linearity and peak-detection are not justified. Peak-detection may hold as an approximation if either the noise fluctuates very slowly compared to the course of the mean activity caused by the stimulus, or if the noise fluctuates fastly, and the internal threshold is high compared to the maximal value of the mean activity and the variance of the noise process. If neither of these conditions is met the firm belief in peak-detection leads to the conclusion that the nonlinearities of the visual system have to be taken into account even for threshold contrasts (Rashbass 1970, Kelly and Savoie 1978). However, what appears to be an *essential nonlinearity* (Kelly and Savoie 1978) may equally well be an effect of TPS.

A popular approach to detection by TPS makes use of the Weibull-distribution and will henceforth be referred to as *Weibull approximation*. This approximation has been suggested by a number of authors (Broekhuysen et al. 1976; Rashbass 1976; Watson and Nachmias 1977; Watson 1979). The motivation behind our search for alternative approximations stems from the fact that for reasons that will be elaborated in greater detail later the Weibull-approximation as proposed by the above mentioned authors is unsatisfactory.

There exist no general, closed expressions for the probability of a level-crossing by a non-stationary stochastic process. Summaries of attempts to find at least approximations for the probabilities of such crossings may be found in Cramér and Leadbetter (1967), Blake and Lindsay (1973) and Leadbetter et al. (1983). However, none of these attempts suits the demands of psychophysics. The two approximations that will be presented here appear to work well. One is due to Hüsler[1] (1986) and is based on results from extreme value theory; Hüsler follows the classical line of attack-

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ing the problem of level-crossings by chopping the interval $[0, T]$ into sub-intervals and deriving the distribution of the maximum of the maxima within these sub-intervals. The other is due to Ditlivsen (1971) who arrived at his approximation in quite a different way: he considered the probability of a level-crossing in a sub-interval given that a level-crossing did not occur in the preceding sub-interval. The relative merits of the two approximations will be discussed. In any case our computations suggest that the noise is fluctuating very fast, indicating that for all practical purposes it may be considered as "white".

2 Detection processes

2.1 Basic notions

The activity in visual channels as generated by some stimulus pattern depends upon (i) the stimulus contrast or intensity m, (ii) the values r of parameters characterising specific features of the pattern, like its duration or temporal frequency.

A trial will be represented by the closed interval $[0, T]$, where T represents the duration of the trial (not of the stimulus presentation). Let $M \subset \mathbb{R}$ be the set of contrasts from which the values for m are chosen. Further, let $Q \subset \mathbb{R}$ be the set of parameter values considered. Such a parameter may be the duration of the stimulus, its spatial or temporal frequency, etc. Let $s(m, r)$ be a stimulus pattern employed in the experiment. For each $r \in Q$, the mapping $P_r : M \to (0, 1)$ will be called *psychometric function*; $P_r(m) \in (0, 1)$ is the probability of a detection response given the stimulus pattern is defined by $s = m \tilde{S}_r$, $\tilde{S}_r$ the function defining the pattern. The set $\mathscr{H} = \{P_r ; r \in Q\}$ will be called a *psychometric family*, this notion was introduced by Falmagne (1982) and turns out to be of importance in the present context because properties of the detection process may be inferred from properties of classes of psychometric functions. For the rest of the paper $\max_t f(t)$ will denote the maximum of the function $f(t)$ over $J = [0, T]$.

Assumptions: (i) The activity of a detecting channel is supposed to be representable by a stochastic process $V_t = \{V(t); t \in [0, T]\}$; (ii) $V(t) = g(t) + \xi(t)$, where $\xi(\cdot)$ is a trajectory of a stochastic process ξ_t, representing stationary noise with $E(\xi(\cdot)) = 0$ for all $t \in [0, T]$, and $g(t)$ represents the mean activity of the channel at time $t \in [0, T]$ for the given stimulus pattern. (iii) There exists an internal threshold with value S such that the stimulus is detected if $X = \max_t(V(t)) > S$.

Definition 1: If the assumptions (i) to (iii) hold, detection is said to be by temporal probability summation (TPS).

g is a deterministic function of t (time) depending upon parameters characterising the system as well as upon m and r. X may be considered a random variable and represents the maximum value of the activity during a

given trial. The probability of detection may now be expressed with reference to the distribution of the random variable X as

$$P_r(m) = 1 - (1 - \gamma)P(X \leqslant S), \tag{2.1}$$

where γ is the probability of guessing that a stimulus was presented in a blank trial. The probability $P_0 = P(X > S | m = 0)$ will be called the *probability of a false alarm*. This term may apply more adequately to $P_r(0)$; however, our definition of the term serves to simplify our language within our context.

The explicit definition of peak-detection will be postponed until Sect. 2.3.

2.2 Approximations for $P(X \leqslant S)$

The approximations to be presented depend upon two parameters, the *internal threshold* S and the *second-spectral moment* λ_2, reflecting the speed of fluctuation of the noise. The meaning of λ_2 may be explicated as follows. Let $R(t)$ be the autocorrelation function of the noise ξ_t allowing for the representation $R(t) = 1 - \lambda_2 t^2/2 + o(t^2)$ for $t \to 0$ which may be considered as the first terms of a Taylor-series. Then, formally, $\lambda_2 = -R''(0)$, with $R''(0)$ the second derivative of R at $t = 0$. If $R(t)$ is sharply peaked at $R(0)$ the random variables $\xi(t)$ and $\xi(t + \epsilon)$ for $\epsilon > 0$ representing the values of the noise at times t and $t + \epsilon$ will not be highly correlated even if ϵ is small and consequently the trajectories of ξ_t will fluctuate fastly; we will then expect a large value for λ_2. Correspondingly, for λ_2 small $R(t)$ will be flat, so that there is a high correlation between neighbouring values of $\xi(t)$, i.e. the $\xi(t)$ fluctuate slowly.

a. Hüsler's approximation. The interval $J = [0, T]$ may be divided into n disjoint intervals J_i of length Δt such that $T = n\Delta t$, or $\Delta t = T/n$. For the ith interval J_i let $X_i = \max_{t \in J_i} V(t)$, $V(t)$ a trajectory of V_t. Then $X \approx \max(X_1, \ldots, X_n)$, so that the distribution function of X is that of an extreme value of the X_i. Clearly, the X_i will not be independent in general, and it is the dependencies among the X_i that cause the difficulties in deriving the distribution function $P(X \leqslant S)$ of X.

Let $R(t)$ be the autocorrelation function of the noise process ξ_t. To tackle the problem of finding at least an approximation for $P(X \leqslant S)$ we may make use of the notion of asymptotic independence. A detailed explication of this concept cannot be given here, see Leadbetter et al. (1983). Roughly, asymptotic independence means that

$$P\left(\bigcap_{i=1}^{n} X_i \leqslant S\right) \approx \prod_{i=1}^{n} P(X_1 \leqslant S)$$

provided that $R(t) \to 0$ for $t \to \infty$ (and consequently $T \to \infty$) and $S \to \infty$ in an appropriate way. We present now a theorem derived by Hüsler (1986):

Theorem 1: *Suppose that ξ_t is a wide-sense stationary Gauss process with $E(\xi(t)) = 0$, $E(\xi^2(t)) = 1$ and auto-correlation function R satisfying $R(t) = 1 - \lambda_2 t^2/2$*

$+ o(t^2)$ as $t \to 0$, $\lambda_2 = -R''(0) < \infty$. Then, for sufficiently high value of S,

$$P(X \leqslant S) \approx \exp\left(-\frac{\sqrt{\lambda_2}}{2\pi} \int_0^T \exp[-\tfrac{1}{2} \cdot (S - g(t))^2] \, dt \right).$$

(2.2)

The proof of the theorem may be found in the Appendix; it is of interest when compared with the derivation of the Weibull approximation. Obviously, $P(X \leqslant S)$ is related to the double exponential distribution $\exp(-\exp(-x))$; this is the limiting distribution for extremes of independent Gaussian random variables.

b. *Ditlivsen's approximation.* A detailed presentation of Ditlivsen's derivation is beyond the scope of this paper, we just sketch the main path of his argumentation.

Let us introduce the abbreviation

$$a(t) := S - g(t) .$$

(2.3)

The probability of not detecting the stimulus is given by $P(X \leqslant S) = P(\xi(\tau) \leqslant a(\tau), \forall \tau \in [0, T])$. Let us define the probability

$$\Gamma(x, t) := P(\xi(\tau) \leqslant a(\tau), \quad \forall \tau \in [t, t + x]) .$$

(2.4)

Clearly, for $t = 0$ and $x = T$ we have $\Gamma(T, 0) = P(X \leqslant S)$. Ditlivsen starts with the known fact that for short intervals $[t, t + x]$ the probability of a level-crossing may be approximated via the joint density function f of $u := \xi(\tau)$ and $v := \xi(\tau + x)$:

$$\Gamma(x, t) = P(\xi(t) \leqslant a(t) \cap \xi(t + x) \leqslant a(t + x)) + o(x)$$

$$= \int_{-\infty}^{a(t)} \int_{-\infty}^{a(t+x)} f(u, v) \, du \, dv \times o(x) ,$$

with $o(x)/x \to 0$ as $x \to 0$.

(2.5)

The problem is then to relate (2.5) to (2.4) in particular for $t = 0, x = T$; here, we just present the result:

Theorem 2: *Let ξ_t be a wide-sense stationary Gauss process with $E(\xi(t)) = 0$ and $E(\xi^2(t)) = 1$ for all $t \in [0, T]$, and suppose the autocorrelation function of ξ_t satisfies $R(t) = 1 - \lambda_2 t^2/2 + o(t^2)$ as $t \to 0$. Suppose further that (2.5) holds (and $\Gamma(x, t)$ is defined as in (2.4)). Let $a'(t) = da(t)/dt = -dg(t)/dt = -g'(t), f(u) = \exp(-u^2/2)/\sqrt{2\pi}$ the Gaussian density function, and $\Phi(u) = \int_{-\infty}^{u} f(t) \, dt$ the corresponding distribution function, and $\gamma = \sqrt{\lambda_2} = \sqrt{-R''(0)}$, λ_2 the second spectral moment of ξ_t. Then*

$$P(X \leqslant S) = \Phi(S) \exp\left[-\int_0^T \left[\gamma f\left(\frac{g'(t)}{\gamma}\right) \right. \right.$$

$$\left. \left. + \Phi\left(\frac{g'(t)}{\gamma}\right) g'(t) \right] \frac{f(a(t))}{\Phi(a(t))} \, dt \right. .$$

(2.6)

A proof of this theorem will not be given here for lack of space; a sketch of the proof of the theorem will be made available on request (Mortensen and Suhl 1990).

The validity of (2.6) has been evaluated by Ditlivsen

(1971) by simulation studies; the expression (2.6) provides excellent approximations for the level crossing probabilities considered here.

c. *Some properties of Hüsler's and Ditlivsen's approximations.* The assumption that detection by TPS is detected by the crossing of a (threshold) level S implies that two parameters, S and λ_2, have to be estimated.

If $m = 0$ then $g(t) = 0$ for all $t \in [0, T]$. If we let T_0 denote the duration of a blank trial we have

$$P_0 = 1 - \exp(-A),$$

(2.7)

with $A = T_0 \sqrt{\lambda_2} \cdot \exp(-S^2/2)/2\pi$ for Hüsler's and

$$A = T_0 \sqrt{\lambda_2} \cdot \exp(-S^2/2)/(2\pi\Phi(S)) + \log \Phi(S)$$

for Ditlivsen's approximation. Hüsler's and Ditlivsen's expressions for P_0 become equivalent for $\Phi(S) \approx 1$. If we let $\lambda_2 \to 0$ in Hüsler's approximation we find $P_0 \to 0$, while from Ditlivsen's expression we find $P_0 \to 1 - \Phi(S) > 0$ if $\Phi(S) < 1$.

For $P_0 > 0$ the expressions for A in (2.7) show that then S and λ_2 cannot be chosen independently of each other. As may be seen from the expressions for A a given value of $P_0 > 0$ is not tied to a *particular* pair of values for S and $\lambda_2 = \lambda_2(S)$; which combination of values for S and λ_2 is the most adequate one has to be determined by other aspects of the data. Note that the relation between S and λ_2 is strictly increasing.

Although the approximations of Hüsler and Ditlivsen appear to be quite different it turns out that they lead to equivalent estimates for S and $\lambda_2(S)$ for given values of P_0 for sufficiently high values of S. For lower values of S estimates based upon Hüsler's approximation become less reliable because the approximation is based upon extreme value statistics. Therefore it is valid only for $S \to \infty$, i.e. for $S > 3$, say, since for a standard Gauss-variable the probability of exceeding the value $S = 3$ is already relatively small. Ditlivsen's approximation is applicable for smaller values of S as well, however at the price of higher computing costs compared with those associated with Hüsler's approximation.

d. *Weibull approximations.* We briefly consider the Weibull approximation for detection by TPS mentioned in Sect. 1 and currently held under high esteem by vision researchers. The expression is based upon the Weibull distribution. As in Hüsler's approximation the interval $[0, T]$ representing a trial is subdivided into n subintervals of length Δt such that $T = n\Delta t$. For the ith interval the probability of *not* detecting the stimulus is assumed to be given by $q(t_i) = \exp(-|g(t_i)|^\beta)$, where β is a free parameter meant to reflect the "effect of noise". g depends upon the parameters m and r. Then detection in the different intervals is postulated to be stochastically independent, so that the probability of detection is given by $P_r(m) = 1 - (1 - \gamma) \times \exp(-\Sigma_i |g(t_i; m, r)|^\beta)$, where γ is again the probability of guessing that a stimulus was presented in a blank trial. Then a passage to the limit, with $n \to \infty$, $\Delta t \to 0$, is

postulated, and the result

$$\mathbf{P}_r(m) = 1 - (1 - \gamma) \exp\left(- \int_{-\infty}^{\infty} |g(t)|^\beta \, dt \right) \quad (2.8)$$

is announced (Broekhuysen et al. 1976; Rashbass 1976; Watson et al. 1977; Watson 1979, 1982). Blommaert and Roufs (1987) replace the integration interval $(-\infty, \infty)$ by $[0, T]$.

There are several difficulties associated with (2.8): *(i)* it is not clear why in a given subinterval the probability of not detecting should be given by $q(t_i) = \exp(-|g(t_i)|^\beta)$. The fact that the Weibull distribution has its merits in the theory of reliability of materials cannot yet be a sufficient reason for its choice. The detection process itself is not specified, i.e. it is not clear whether the detection process is assumed to be a level-crossing or to be of some other nature; *(ii)* the process of passing to the limit is rather obscure, it remains unclear why for $n \to \infty$, $\Delta t \to 0$ the $q(t_i)$ can simply be replaced by $q(t)$. The assumption of stochastic independence of detection events in neighbouring intervals will not hold for $\Delta t \to 0$ and $n \to \infty$ unless the underlying noise process is assumed to be white noise. However, it is by no means clear why we should *assume* white noise a priori, and so the dependencies among the "noise"-fluctuations at times t and $t + \epsilon, \epsilon > 0$, are not adequately dealt with. *(iii)* For $m = 0$ the probability P_0 equals zero; false alarms are due to guessing only. This may be so in certain experiments, but modelling detection by TPS should at least allow for the possibility of "true" false alarms (cf. Nachmias 1981).

A different derivation of the Weibull approximation was provided by Maloney and Wandell (1984); their work may be criticised along similar lines.

2.3 Peak-detection

We now provide a formal definition of peak-detection in order to avoid ambiguities that appear to be connected with this concept.

Definition 2: Let $g(t; m, r)$ denote the mean-value function of the stochastic process representing the activity of the detecting system, with stimulus parameters $m \in M$, $r \in Q$, and let $g_0 = \max_t g(t; r)$ be representable by a strictly increasing mapping $g_0: M \to \mathbb{R}$. For fixed $r \in Q$ let $G_r = g_{0r}(M) \subset \mathbb{R}$. Suppose that for $r \in Q$ the psychometric functions are representable by strictly increasing mappings $P_r: G_r \to (0, 1)$, i.e. $P_r(m) = P(g_{0r})$. Then detection is said to be *peak-detection*, and the psychometric family $\mathcal{H} = \{P_r; r \in Q\}$ will be said to be a *peak-detection family*.

The definition of peak-detection given here does not contain any reference to the noise process. This lack is also found in the usually less explicit definitions found in the literature (e.g. Roufs 1974; Kelly and Savoie 1978; Roufs and Blommaert 1981). A characterisation of the noise compatible with peak-detection will be given, together with some further implications of peak-detection, in Theorem 3; to prepare the presentation of the theorem we introduce the following concept:

Definition 3: Let, for given $r \in Q$, $m_0 = m_0(r)$ be the value of $m \in M$ such that $P_r(m_0) = p_0$ a certain constant, say $p_0 = 1/2$. Let $\chi[m_0(r)] = dP_r(m)/dm|_{m = m_0}$ be the slope of $P_r(m)$ at $m = m_0(r)$. Then

$$q_r := 1/[m_0(r)\chi(m_0(r))\sqrt{2\pi}] \quad (2.9)$$

will be called the *generalised Crozier quotient*.

If the psychometric function can be expressed in terms of a Gaussian distribution function it is easily shown that q_r as defined in (2.9) equals the ratio $\hat{\sigma}_r/\mu_r$, where $\hat{\sigma}_r$ is the standard deviation of the Gaussian density of m and μ_r is the corresponding expected value; in this form the quotient q_r was originally introduced by Crozier (1935/1936). If the psychometric function P_r is indeed given by a Gaussian distribution function $\hat{\sigma}_r$ may be determined from the slope of P_r in m_0. Crozier found that the ratio q_r is practically constant. This is known as *Crozier's law*, and σ_r/μ_r is known as *Crozier's quotient* (LeGrand 1968, p. 269). We will speak of Crozier's law if q_r as defined in (2.9) equals a constant on Q *without* assuming that the psychometric function is necessarily Gaussian. In any case can the slope of P_r be determined empirically and consequently q_r may be computed for any empirically determined psychometric function without any specific assumption about P_r. Now most of the published values of the Crozier quotient are based on the Gaussian assumption and have been computed according to (2.9) (e.g. Blackwell 1963), and so we will compute the quotient analogously although our psychometric functions are not Gaussian. A discussion of some implications of Crozier's law may be found in LeGrand (1968).

Definition 4: Let $\mathcal{H}$ be a psychometric family whose members are parallel when plotted on a $\log(m)$-scale. Then the $P_r \in \mathcal{H}$ will be called *log-parallel* and h will be called a *log-parallel family*.

Let $x = \log(m)$ and let $F_r(x) := P_r(e^x)$, $F_s(x) := P_s(e^x)$, $r \in Q, s \in Q$, be two psychometric functions that are parallel on the $\log(m)$-scale. This means that there exists a constant $\alpha(r, s)$ independent of x, with $F_r(x) = F_s(x + \alpha(r, s))$ for all x. With $\alpha(r, s) = \log[b(r, s)]$ we have then $P_r(m) = P_s[m \cdot b(r, s)]$, i.e. any two psychometric functions $P_r(m)$ and $P_s(m)$ from $\mathcal{H}$ differ with respect to a factor $b(r, s)$ of the independent variable m. We may generally write $P_r(m) = P(ma(r))$ with $a(r) = b(r, s_0)$ for a certain reference value s_0.

The purpose of the following theorem is to make explicit the implications and predictions of peak-detection in the strict sense of Definition 3. The data are indicative of detection by TPS when these implications cannot be observed.

Theorem 3: *Let $\mathcal{H}$ be a psychometric family such that each $P_r \in \mathcal{H}$ is differentiable with respect to $m \in M$ and $r \in Q$. Then the following statements hold:*

(i) Suppose that the stimulus is detected if $X = \max_t(g(t) + \xi(t)) > S$. Then detection is peak-detection as defined in Definition 3 if and only if the trajectories ξ of the noise process ξ_t are constants (almost surely,

i.e. trajectories that are not constants have probability measure zero) varying randomly only between trials.

(ii) If $\mathscr{H}$ is a log-parallel psychometric family and if the $P_r \in \mathscr{H}$ are strictly increasing with $m \in M$, then Crozier's law holds for $\mathscr{H}$.

(iii) Let $\mathscr{H}$ be a peak-detection-family and let the detecting channel be linear; then $\hbar$ is also a log-parallel psychometric family.

The proofs of the statements may be found in the appendix. Statement *(i)* characterises the "noise". One may say that in case of peak-detection the noise consists of a random shift of the internal threshold according to some distribution function; within the trial $[0, T]$ the threshold remains constant. As indicated above this is certainly an idealisation. Consequently the statements *(ii)* and *(iii)* will be reflected in empirical data at best in an approximative sense. Since peak-detection (in the strict sense of Definition 3) implies that the noise trajectories are constant during a trial one condition under which the assumption of peak-detection may be employed as an approximation is characterised by a small value of λ_2, such that the fluctuations are slow compared to the changes of g.

Let us briefly consider Hüsler's and Ditlivsen's approximation for the case $\lambda_2 \to 0$. In this case Ditlivsen's approximation converges towards the expression for peak-detection in the strict sense. The case $\lambda_2 \to 0$ is not covered by Hüsler's approximation: it follows from (2.2) that for $\lambda_2 \to 0$ $P(X \leqslant S) \to 0$ regardless of which $g(t)$ is generated by the stimulus presentation. Hüsler's approximation is no longer meaningful for small values of λ_2 because it rests on the assumption of asymptotic independence of neighbouring values of the $\xi(t)$ (see the proof of (2.2)); the approximation is therefore applicable only if λ_2 assumes greater values. This does not mean that Hüsler's approximation cannot be used if peak-detection holds as an approximation: from Definition 2 one may also conclude that the peak-detection approximation may hold if an arbitrary, in particular high value of λ_2 accompanied with a high value of S is given, such that a level-crossing can occur only in the vicinity of the maximum of g.

Statements *(ii)* and *(iii)* imply that if detection is peak-detection, then Crozier's law holds. However, Crozier's law does not, in general, imply peak-detection and therefore not log-parallelity. A violation of Crozier's law implies a lack of log-parallelity and points to detection by TPS. Log-parallelity and therefore the validity of Crozier's law, on the other hand, does not necessarily exclude detection by TPS; for instance, psychometric functions defined in terms of the Weibull distribution as in (2.8) respresent a log-parallel psychometric family. Note that constant Crozier quotients *do not* imply constant slopes of the psychometric functions. Further, a value of $P_0 > 0$ is compatible with peak-detection in the sense of Definition 3 and therefore not yet indicative of detection by TPS. Rather, TPS is indicated by a violation of Crozier's law or, even better, by a lack of log-parallelity of psychometric functions. We introduce therefore

Definition 5: The violation of Crozier's law and, consequently, of log-parallelity of psychometric functions will be called the *effect of temporal probability summation.*

3 Application to detection data

3.1 Estimation of S and λ_2 from flicker detection data

Let the stimulus be a sinusoid that is gated in order to avoid transients. Let f be the temporal frequency and k the number of complete cycles (Fully Fledged Cycles, FFC), and suppose that for each element of a set $\{(f, k)\}$ the corresponding threshold modulation $m_k(f)$ is determined. The data from such an experiment, which was actually carried out by Roufs (1974), may be employed to estimate the values of S and λ_2.

To begin with, suppose that peak-detection in the strict sense, i.e. $\xi(\cdot) = \zeta$ during a trial of length less than some critical duration T_0, holds. Then the value of $m_k(f)$ for different values of k, such that the total duration of the stimulus presentation is less than T_0, should be the same, since then the probability of detection depends only upon $\max_t g(t)$. However, this prediction does not correspond to Rouf's finding, which may be summarized as

$$\log m_k(f) = \beta \cdot \log k + \mu, \tag{3.1}$$

with $\beta < 0$ independent of f, and $\mu = \mu(f)$. This may be interpreted as indicating somehow the effect of temporal probability summation.

Hüsler's and Ditlivsen's approximation are of the form $P(X > S) = 1 - \exp(-\int_0^T \phi(t) \, dt)$. ϕ depends upon the parameters S and λ_2. Here we identify, for given value of the frequency f, the parameter r with the number of cycles, i.e. $r = k$. Now $P_k(m_k(f)) = p_0$ a constant iff $\int_0^T \phi(t; S, \lambda_2) \, dt = I_0$ a constant depending upon the value of p_0 (i.e. $I_0 = -\log(1 - p_0)$). Recall that for a linear system with constant coefficients the response to $m \cdot \sin(\omega t)$ is given by $g(t) = m|H(\omega)| \sin(\omega t + \varphi(\omega))$. Because the stimuli were gated we may neglect the onset and offset parts of the responses and identify *(i)* $t = 0$ with the start of the first full cycle, and *(ii)* $t = T$ with the end of the last full cycle. Thus we get rid of the problem of having to find an estimate of the phase shift $\varphi(\omega)$ so that we may put $g(t) = m_k(f)|H(\omega)| \sin(\omega t)$, with $\omega = 2\pi f$. Since $\phi(t) = \phi(t + 2\pi)$ and the duration of a single cycle equals $1/f$ we have $\int_0^T \phi(t) \, dt = k \int_0^{1/f} \phi(t) \, dt = k \int_0^{1/f} \phi(m_k|H(\omega)| \sin(\omega t)) \, dt$. Employing the substitution $u = \omega t$ we have, since $\omega = 2\pi f$, for the integral

$$I_0 = \int\limits_0^T \phi(t) \, dt$$

$$= \frac{k}{\omega} \int\limits_0^{2\pi} \phi[m_k(f)|H(\omega)| \sin u; S, \lambda_2] \, du. \tag{3.2}$$

The values of $m_k(f)$ are computed from (3.1), using the empirically determined values of β and μ. Clearly, the values of $\phi(t; S, \lambda_2)$ in (3.2) can be computed only if

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the values of $|H(\omega)|$ are given. Usually, the reciprocal threshold values are taken as estimates for $|H(\omega)|$; however, these estimates are based on the assumption of peak-detection, and we do not know at this stage whether the assumption is justified. Consequently, the $|H(\omega)|$ have to be estimated alongside with S and λ_2. The values of S and λ_2 have to satisfy the following conditions:

(i) They have to be related to the value of P_0 by (2.1), depending upon which approximation is chosen.

(ii) For the chosen S and λ_2 estimates should exist for $|H(\omega)|$ that are constant for all values of the number k of cycles, for a given value of f.

(iii) The Crozier-quotients q_r computed for the theoretical psychometric functions should be about constant, i.e. for all r, $q_r \approx \bar{q}$ and the value of $\bar{q}$ is specific for the subject whose data are considered.

Generally, the duration T_0 of a blank trial was taken to equal 2 s. The empirical Crozier quotients $\hat{q}(f)$ for the different temporal frequencies f satisfied the condition $\hat{q}(f) \approx .25$ for all f, meaning that the differences between empirical Crozier quotients for different values of f are so small that they enter only at the third position after the decimal point at best. Therefore we took $\hat{q}(f) = .25$ as the value to be approximated by the theoretical Crozier quotients, i.e. by the Crozier quotients associated with the psychometric functions defined in terms of Hüsler's and Ditlivsen's approximations.

We considered the data from two subjects. For subject JP the estimate for β in (3.1) was $\hat{\beta} = -.22$ for all temporal frequencies, and $\hat{\mu}_1 = -1.82$ for $f_1 = 8$ Hz, $\hat{\mu}_2 = -1.16$ for $f_2 = 22.5$ Hz and $\hat{\mu}_3 = 0$ for $f_3 = 40$ Hz. For subject JAJR[2] the estimate for β was $\hat{\beta} = -.44$, and the $\hat{\mu}$-values were about the same as for subject JP; their exact values are unimportant since the effect of TPS is indicated by the value of β (peak-detection in the sense of Definition 3 would imply $\beta = 0$).

Empirical values of P_0 were not given to us. This did not turn out to be a serious drawback, since the demands (ii) and (iii) even imply an estimate of P_0. To explore the effect of the value of P_0 upon the estimates of λ_2, $S(\lambda_2)$ and of $|H(\omega)|$, the parameters S and λ_2 were estimated for different values of P_0, employing Hüsler's as well as Ditlivsen's approximation. It was found that (i) values of S and λ_2 can be found such that the corresponding value for $|H(\omega)|$ were practically constant for all values of k, and (ii) Crozier quotients in the neighbourhood of .25 could only be found for values of P_0 not larger than .02 (for $P_0 > .03$ the requirement $q_r \approx .25$ implies the need for values of S and λ_2 not allowing the integration procedure to converge; these values of S and λ_2 appear to be meaningless).

For the slope $\hat{\beta} = -.22$ of the regression line (3.1) one arrives at practically identical results with both approximations; they are summarised in Table 1.

Obviously, the differences of the estimates for S are negligible, and the estimates for λ_2 are qualitatively equivalent: they appear to reflect a noise process ξ_t that may be considered for all practical purposes as white.

For all three temporal frequencies values of $|H(\omega)|$ approximately constant for the different values of the number of cycles and correspondingly constant values of the Crozier quotients, for both values of P_0, were found employing the two approximations. The estimates for $|H(\omega)|$ and the Crozier quotients from Ditlivsen's approximation may be found in Fig. 1(a); the corresponding graphs for Hüsler's Approximation are almost identical.

The side condition $q(f) \approx .25$ was satisfied to a greater extent for the smaller value of P_0, $P_0 = .01$. This means that for given value of β a smaller value of P_0 implies parameter values S and $\lambda_2(S)$ for which the peak-detection approximation is better than for a bigger value of P_0. Recall that, on the other hand, the value of P_0 per se is not yet counterindicative of peak-detection, since in case of peak-detection the value of P_0 depends upon the variance of the random variable ζ and the value of h_0 (the maximum of $h(t)$, the unit response of the detecting channel). So the value of P_0 is indicative of an effect of TPS only in conjunction with that of β and with the values of the Crozier quotients. Note further that for a given value of β a higher value of P_0 implies not only a higher value of λ_2 in order to achieve a fit to the data, but also a higher value of S. An increase of λ_2 without an increase of S, or, alternatively, a decrease of S without a change of the λ_2-value is not compatible with the data.

Let us recall that according to Theorem 3, (ii) and (iii) peak-detection by a linear channel implies that Crozier's law holds, i.e. the Crozier quotients q_r corresponding to different values of the number k of cycles, should be constant. Therefore, if the q_r differ from one another detection will not be peak-detection in the strict sense of Definition 3. Note that the Crozier quotients for the frequency $f = 8$ Hz show a greater variation with k than the Crozier quotients for $f = 22.5$ Hz. This reflects the intuitively very plausible fact that for lower frequencies the duration of the stimulus presentation is longer, and an increased stimulus duration goes with an increased chance of a level crossing (recall that the expressions (2.2) and (2.6) for $P(X \leqslant S)$ depend upon the duration T of a trial). Furthermore, for low frequencies the deterministic response $g(t) = m|H(\omega)| \sin(\omega t + \phi(\omega))$ is more spread out in the neighbourhood of its maximal value $|H(\omega)|$, so that again the chance of a level crossing in the neighbourhood of the maxima is greater than for higher

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Table 1. Estimates for S and λ_2

P_0	Hüsler		Ditlivsen	
	S	λ_2	S	λ_2
.01	4.772	506095.75	4.451	400193.39
.03	7.562	6.256298E + 22	7.750	7.06218E + 22

a

Ditilvsen's approximation; Po = 0.01

Ditilvsen's approximation; Po = 0.03
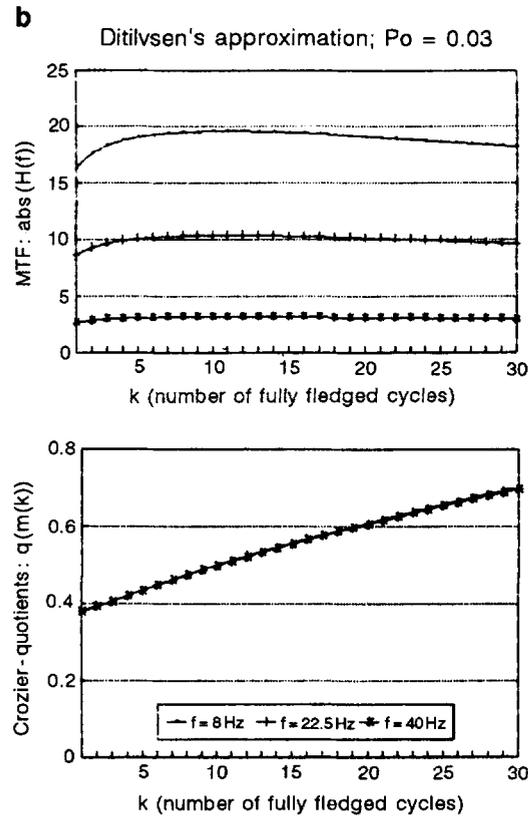
b

Ditilvsen's approximation; Po = 0.01

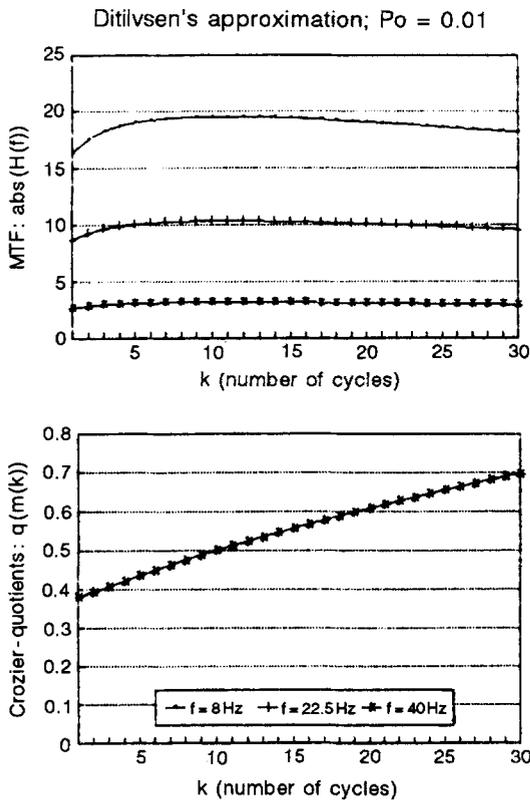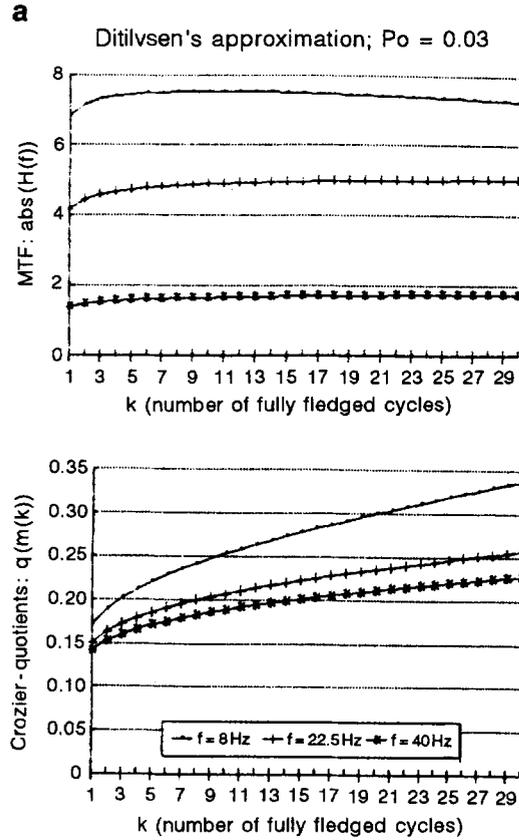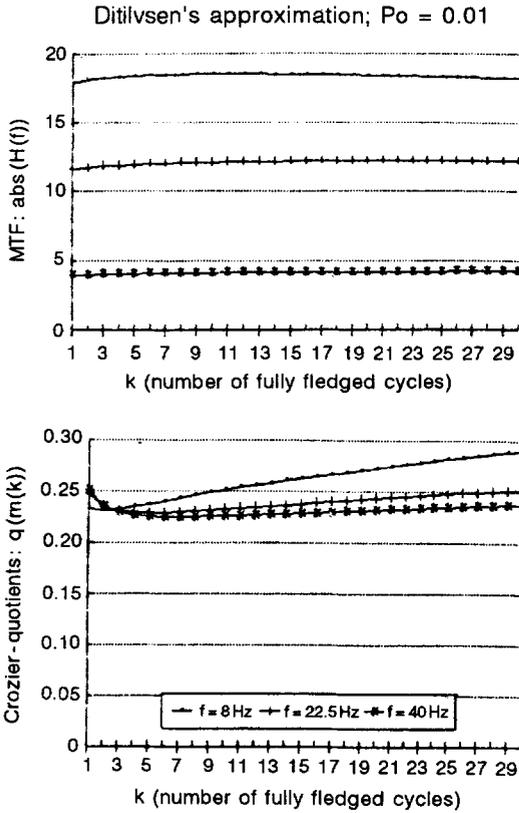Ditilvsen's approximation; Po = 0.03

Fig. 1a, b. Modulation transfer functions (MTFs) and Crozier quotients derived from the Fully-Fledged-Cycles data; **a** $\beta = = .22$ (cf (3.1)), **b** $\beta = -.44$; see text for further details

frequencies. So for lower frequencies the effect of TPS is altogether greater than for higher frequencies. Note further that for $P_0 = .01$ the estimates of $|H(\omega)|$ are practically constant also for smaller values of k ($k < 5$), in contrast to the estimates for $P_0 = .03$. Further, the estimates for $|H(\omega)|$ are *higher* for the smaller value of P_0.

Let us have a look at the estimates for S and λ_2 for $\hat{\beta} = -.44$. For this value of $\hat{\beta}$, Hüsler's approximation *does not* yield estimates for S and λ_2 that allow for constant values of $|H(\omega)|$, because these data require comparatively small values for the parameter S. Thus the condition of "large" values for S required for the validity of Hüsler's approximation, contained in the extreme value approach taken for its derivation, is violated. Figure 1(b) shows the estimates for $|H(\omega)|$ and the Crozier quotients, based upon Ditlivsen's approximation. For (a) $P_0 = .01$ we found $S = 2.651$, $\lambda_2 = .40134$, and for (b) $P_0 = .03$ we have $S = 2.654$, $\lambda_2 = 7.852$. Again, the higher value of P_0 requires higher values of S and $\lambda_2(S)$; however, for this value of $\hat{\beta}$ the increase of S with P_0 is quite small compared with the case $\hat{\beta} = -.22$. Note that the higher value of P_0 implies a higher value in particular of λ_2, and not so much of S.

Note that the case $\hat{\beta} = -.44$ requires a *smaller* value of S than the case $\hat{\beta} = -.22$, and a correspondingly *smaller* value of λ_2. Detection is peak-detection in the strict sense if $\lambda_2 = 0$, so one could argue that the smaller the value of λ_2, the better the assumption of peak-detection holds as an approximation. However, it was not possible to achieve estimates for S and λ_2 allowing for approximately constant Crozier quotients. (Note that the Crozier quotients for $\hat{\beta} = -.44$ are *higher* in the mean than for $\hat{\beta} = -.22$.) This lack of constancy points, according to Theorem 3, to detection by TPS. Indeed, the "small" values of λ_2 found for $\hat{\beta} = -.44$ go with "small" values of S, and since the relation between S and λ_2 is nonlinear the *combination* of small values for S and λ_2 may imply detection by TPS.

3.2 Peak-detection as an approximation

The implication of peak-detection is *constancy* of Crozier quotients *and* log-parallelity of psychometric functions. The value of the slope of psychometric functions is not indicative of detection by TPS, as may be readily seen from (2.1): with $X = g_0 + \zeta$, ζ a random variable that is constant within $[0, T]$; the slope of $P_r(m) = 1 - (1 - \gamma)P(X \leqslant S)$ depends upon the value of γ, and because of $P(X \leqslant S) = P(\zeta \leqslant S - mh_0)$, $mh_0 = g_0$ upon that of h_0. So even if detection is peak-detection in the strict sense the psychometric function may be steep or flat.

Suppose we have determined psychometric functions for various values r of a stimulus parameter. If detection is not peak-detection in the strict sense of Definition 3 the Crozier quotients will vary, and we may compute their mean value as well as their variance. This variance may be taken as a measure of constancy.

A better test of the hypothesis of peak-detection as an approximation is to test the log-parallelity of the psychometric functions for the different values of the stimulus parameter since (almost) complete psychometric functions contain more information about the detection process than the corresponding Crozier quotients alone. To illustrate the validity of the peak-detection approximation we have thus determined the psychometric functions for rectangular pulses of different durations T_s (T_s being the parameter r according to which the stimuli and therefore the psychometric functions differ) employing the values of S and λ_2 estimated from the Fully-Fledged-Cycles data. We assumed either $P_0 = .01$ or $P_0 = .03$, and plotted the detection probabilities on a log-m scale, see Fig. 2.

If these psychometric functions had been determined empirically we would not hesitate to call them log-parallel and interpret them as not contradicting the hypothesis of peak-detection at least as an approximation in this case. Clearly, the psychometric functions are not *exactly* log-parallel, but exact log-parallelity can of course only be expected for $\lambda_2 = 0$.

Empirically determined psychometric functions may not look that clean, and we have not touched the statistical problems associated with the question how one decides whether observed deviations from log-parallelity are significant or not. In principle, such a test is a relatively straightforward affair if it were not for the difficulties associated with the estimation of the P_r-values; we will not elaborate these matters here.

4 Discussion

We assumed that sensory noise can be represented by a wide-sense stationary Gaussian stochastic process ξ_t with autocorrelation function $R(\tau)$ for which $R''(0)$ exists; $\lambda_2 = -R''(0)$ is known as second-spectral moment and may be taken as a measure of the speed of the fluctuations of ξ_t.

It may now be of interest to elaborate the relation between the value of λ_2 as estimated from psychophysical data as compared with corresponding estimates from neurophysiological investigations. A complete discussion of these matters is clearly beyond the scope of this paper, but it may be worthwhile to point out some open questions.

One problem with such an undertaking results from the question whether we may relate the estimates of λ_2 to the activity of a single cell. Most likely the detection process involves a set of neurons in the visual cortex (cf. Freeman 1990). Therefore estimates of threshold voltages in receptor cells (e.g. Fain et al. 1977) may not be very helpful in this context, also because we are not dealing with absolute thresholds. If we agree that more than one cell is involved in the detection process we have to tackle the question how the activity is monitored or integrated by higher centers of the brain. The work of Roufs and Blommaert (1981) indicates that the role of probability summation is at best negligible here (cf. Mortensen (1988) for a general discussion of
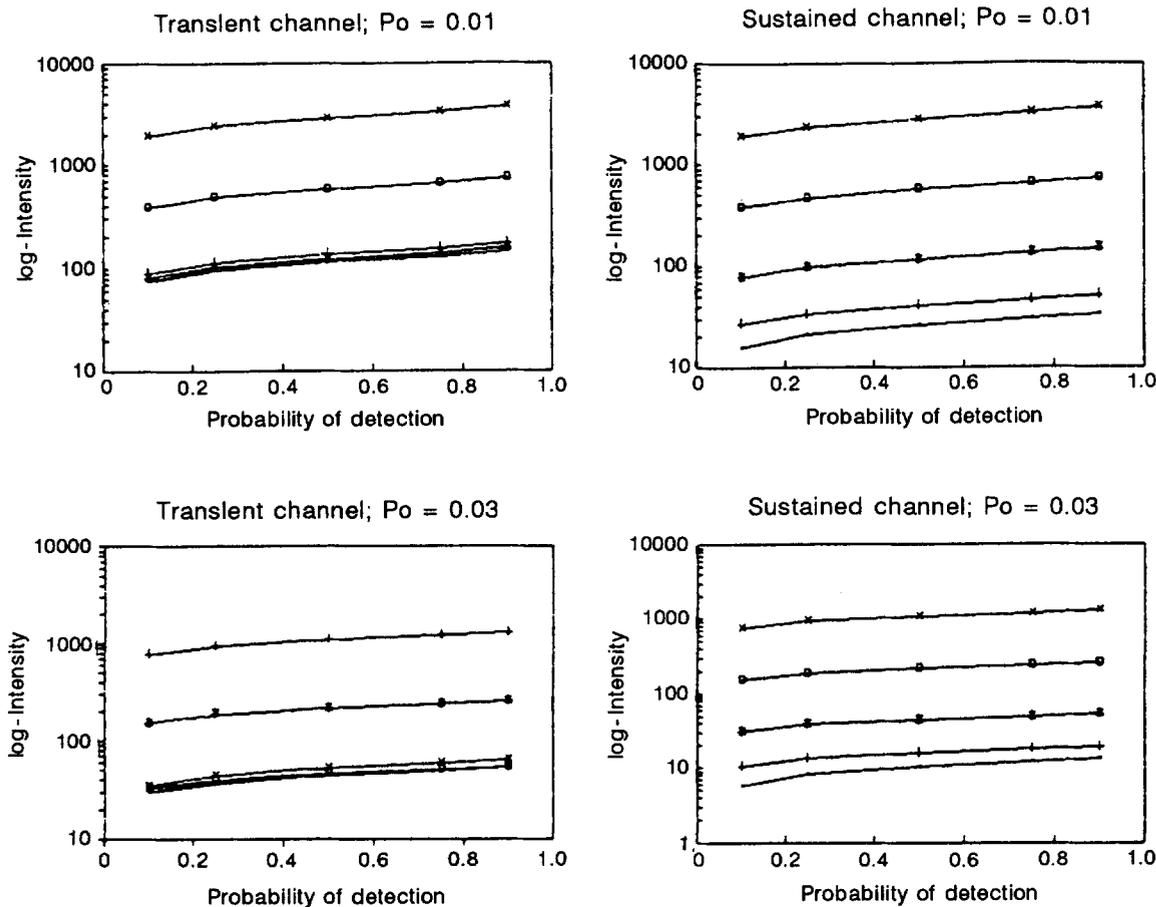
Transient channel; Po = 0.01

Sustained channel; Po = 0.01

Transient channel; Po = 0.03

Sustained channel; Po = 0.03

Fig. 2. Log-parallel psychometric functions (for technical reasons exchange of x-axis (log-intensity) and y-axis (probability of detection); Ditlivsen's approximation. Stimuli: rectangular pulses of various durations $T_s(\log T_s: \cdot 2.8, +2.1, *1.4, \square .7, \times .0)$. Effects of TPS as indicated by deviations from log-parallelisms occur only for the sustained channel

this point). So we may assume some linear form of integration, which *may* be equivalent to considering a single neuron. Clearly, these considerations are highly speculative and require a lot more work for their substantiation.

Neurophysiological work refers closely to molecular processes in the cell membrane (De Felice 1981; Läuger 1984), which implies models of noise that in one way or another, often implicitly, relate noise (or the activity of the cell in general) to Brownian motion or to processes derived from Brownian motion. This again means that one is dealing with Markov processes. For instance, Bevan et al. (1979) consider a process with autocorrelation $R(\tau) = ce^{-\alpha|\tau|}$. If we combine this with our assumption of a stationary Gauss process we are indeed dealing with a Markov process (cf. Papoulis 1965, p. 537), which may be derived as a special type of diffusion process and which is related to Langevin's equation for the velocity of a free particle in Brownian motion. However, for a process characterised by the autocorrelation function $R(\tau) = ce^{-\alpha|\tau|}$ a second-spectral moment does not exist.

Generally, λ_2 as a noise parameter is practically never determined in neurophysiological investigations

(De Felice 1981; Kolb 1991). Further, estimates of the power spectrum $S(\omega)$ for various values of ω (Bevan et al. 1979) or of c and α as parameters of $R(\tau)$ depend upon the type of neuron, its size and temperature, and so the published values of $S(\omega)$ do not allow for an estimate of the corresponding value of λ_2.

So we have excluded Markov processes from our consideration. From a formal point of view Markov processes in the strict sense are idealisations (Honerkamp 1990) so that we may argue that in fact we have covered a very general case. While this is certainly a rather puritan point of view we take, for the time being, our model as a functional description of the visual system, perhaps representing some guidelines for studies relating more closely psychophysical and neurophysiological data.

Appendix: proofs of theorems

Theorem 1 (Hüsler): Let J_i denote the half-open interval $((i-1)\Delta t, i\Delta t]$. $P[\max_t(g(t) + \xi(t)) \leq S] = P[\xi(t) \leq S - g(t), \quad \forall t \in [0, T]] \approx P[\xi(t) \leq S - g(t_i), \quad \forall t \in ((i-$

$1)\Delta t, i\Delta t], i \leqslant n]$ with $t_i \in J_i$. For $T \to \infty$ one has approximately

$$P[\max_t(g(t) + \xi(t)) \leqslant S] \approx \prod_{i=1}^{n} P[X(t) \leqslant S - g(t_i),$$

$$\forall t \in ((i-1)\Delta t, i\Delta t]]$$

$$= \prod_{i=1}^{n} P[M(\Delta t) \leqslant s - g(t_i)]$$

with $M(\Delta t) = \max_{0 \leqslant t \leqslant \Delta t} \xi(t)$. Let $f(z) = \exp(-z^2/2)/\sqrt{2\pi}$ be the Gaussian density and $\Phi(u) = \int_{-\infty}^{u} f(z)\, dz$ be the corresponding distribution function. As shown in Leadbetter et al. (1983, p. 166), there exists for given $\theta < 1$ a $h_0(\theta) \in \mathbb{R}$ such that for $0 \leqslant h \leqslant h_0(\theta)$ the approximation $P[M(\Delta t) > u] \leqslant 1 - \Phi(u) + \mu\Delta t \geqslant 1 - \Phi(u) + \theta\mu\Delta t$, with $\mu = \mu(\Delta t) = \sqrt{\lambda_2} f(u)$. Since θ may be chosen arbitrarily close to 1 and $1 - \Phi(u) = o(\mu(\Delta t))$ for fixed Δt one has thus $P(M(\Delta t) > u) \approx \mu\Delta t$. Thus, with $u_i = S - g(t_i)$,

$$P[\max_t(g(t) + \xi(t)) \leqslant S]$$

$$\approx \prod_{i=1}^{n} [1 - (1 - \Phi(u_i) - \mu(u_i)\Delta t)]$$

$$\approx \exp\left[-\sum_{i=1}^{n} \Delta t \cdot \frac{\sqrt{\lambda_2}}{2\pi} \exp(-u_i^2/2) \right](1 - o(1))$$

$$\approx \exp\left(-\frac{\sqrt{\lambda_2}}{2\pi} \int_0^T \exp(-\tfrac{1}{2}(S - g(t))^2)\, dt \right). \quad \square$$

Theorem 3: *(i) The "if"-part is somewhat trivial: if for all $\xi \in \xi_t$ the condition $\xi = \zeta$ a constant within $[0, T]$ holds, then $P(\max_t(g(t) + \xi(t)) > S) = p(\max_t g(t) + \zeta > S)$ and detection is clearly peak-detection. The "only if"-part is intuitively clear: if the $\xi \in \xi_t$ reflect random fluctuations within $[0, T]$, the shape of g will have an influence upon the probability of detection, contrary to the assumption that the probability of detection depends only upon $g_o = \max_t g(t)$. So the assumption implies that the ξ are constant within $[0, T]$. Explicitly, the proof is as follows: to keep the notation simple we suppress the index r and write g_0 instead of $g_0(m)$. According to Definition 3 the probability of detection is completely determined by the value of the maximum g_0, irrespective of the form of g. So let G_0 be the set of all functions g having, within $[0, T]$, the maximal value g_0. Let Σ_0 be the set of trajectories of ξ_t such that detection occurs for the special case $g(t) = g_o$ for all $t \in [0, T]$; surely, this function is contained in G_0. Let Σ_g be the corresponding set for arbitrary $g \in G_0$. For this g, $P(\max_t(g(t) + \xi(t) > S) = P(g_0 + \max_t \xi(t) > S) = P(\max_t \xi(t) > S - g_0)$. Obviously, $\Sigma_g \subseteq \Sigma_0$. On the other hand, $P(\max_t \xi(t) > S - g_0) = P(\max_t(g(t) + \xi(t) > S)$ for arbitrary $g \in G_0$ if detection is peak-detection, and therefore $P(\Sigma_0) = P(\Sigma_g)$, so that for $\Sigma_{0g} = \Sigma_0 - \Sigma_g$, $P(\Sigma_{0g}) = 0$, for all $g \in G_0$. Further, suppose there exists a subset $\Sigma_0(I) \subseteq \Sigma_0$ of trajectories ξ with $\xi(t) \leqslant S - g_0$ for $t \in I$, I an arbitrary subinterval of $[0, T]$. Then there exist functions $g \in G_0$ assuming the maximal value g_0 within I such that $\xi(t) \leqslant S - g_0$ for all $t \in (0, T)$ and consequently the trajectories from $\Sigma_0(I)$ do*

not belong to the set Σ_g; since $P(\Sigma_0) = P(\Sigma_g)$ it follows that $P(\Sigma_0(I)) = 0$ and we have the finding that for all $\xi \in \Sigma_0$, $\xi(t) > S - g_0$, i.e. $\min_t \xi(t) > S - g_0$, so that for all $g \in G_0$, $P(\max_t(g(t) + \xi(t)) = P(\min_t \xi(t) > S - g_0)$. It follows further that for all $\xi \in \Sigma_0$, $P(\min_t \xi(t) > S - g_0) = P(\max_t \xi(t) > S - g_0)$. So the distribution functions for the random variables $\min_t \xi(t)$ and $\max_t (t)$ are identical, and this means that $\min_t \xi(t) = \max_t \xi(t)$ almost surely (a.s.). Consequently, the trajectories of ξ_t are random variables with constant value within $(0, T)$, a.s.

(ii) Let $\hbar$ be a log-parallel psychometric family. Then there exists a function F such that for all $\mathbf{P}_r \in \hbar$, $P_r(m) = F(m \cdot a(r))$. Let $b = m \cdot a(r)$. Obviously, $\mathbf{P}_r(m) = p_0$ a constant if and only if $b = m \cdot a(r) = b_0$ a certain constant corresponding to p_0. Now, on the one hand, $\mathbf{P}_r$ is a strictly increasing function of m, but since $\mathbf{P}_r$ is completely determined by the value of b it follows that $\mathbf{P}_r$ is a strictly increasing function of b. Thus, $\mathbf{P}_r(m) = p_0$ iff $b = b_0$ a certain constant. Then $d\mathbf{P}_r/db|_{b=b_0} = \beta$ a constant and $d\mathbf{P}_r/dm|_{m=m_0} = \beta a(r) = \chi(m_0)$, with $m_0 = m_0(r)$. Then $m_0 \chi(m_0) = \beta m_0 a(r) = \beta b_0 = c_0$ a constant on Q, and consequently $q_r = $ constant on Q. Note that the linearity of the channel was not used here.

(iii) If $\mathscr{H}$ is a peak-detection family and if the detecting channel is linear, then in particular $\mathbf{P}_r(m) = P(mh_0(r))$ with h_0 the maximum of the unit response of the detecting channel, and h_0 is independent of m. This holds for all $\mathbf{P}_r \in \mathscr{H}$; consequently, $\mathscr{H}$ is a log-parallel psychometric family. $\square$

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